Optimal design of on-demand feeder services with modular autonomous vehicles

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1 Introduction

With the advent of Modular Autonomous Vehicles (MAVs), transit agencies can adapt to fluctuating demand by flexibly *coupling* several MAVs into a Transit Unit (TU) per dispatch. We further envision the benefit of *decoupling* TU into MAVs in On-Demand Feeder Transit (ODFT), which connects a transportation hub and offers door-to-door services to patrons spread over a distant region. The benefit would come from the reduced routing distance/time to visit N points within the service region, traditionally accomplished by a single bus vehicle but now by several MAVs, each covering a subset of N points. However, whether *recoupling* the MAVs into a TU on return trips needs careful investigation since additional delays may be caused by waiting for the last-arrival MAV.

To demonstrate the concept's effectiveness, we propose an optimal design model to determine the key operational features for MAV-based ODFT, such as TU sizes at dispatch, dispatch headways, and zone partitions, which can vary spatially to suit non-uniform demand distributions. The approach of continuum approximation is used to derive analytical expressions of system metrics including patrons' routing time, waiting time, and non-stop line-haul travel time, as well as the agency's operational costs. Closed-form relationships are obtained for the optimal conditions, leading to an efficient solution algorithm. Numerical studies show that MAV-based ODFT consistently outperforms traditional bus-based ODFT (which can be seen as a special case with no coupling/decoupling functions), with generalized system cost savings exceeding 3%. Notably, the advantage of MAV-based ODFT over its counterpart diminishes when the routing distance/time variance is large. This underscores the necessity of advanced operational algorithms to minimize MAV trips' variance and leverage the flexibility of MAVs in practice.

2 Methodology

We consider a rectangular suburb region $\mathbf{R}$ connected to a transportation terminal by a J-km highway, as shown in Figure 1. The transit agency divides the suburb region of dimensions $L \times W$ into multiple service zones with an area of $A(x)$, which may vary over location denoted by a two-dimensional vector $x \in \mathbf{R}$ ($x = (0, 0)$ indicates the entrance/exit of the region).

The transit agency dispatches the MAVs in Transit Units (TU) with the capacity of $S(x)$ composed of $I \in \{1, 2, ..., K\}$ number of MAVs with a capacity of c [seats/MAV] each. Here K is the maximum number of MAVs coupled in one TU. The headway of dispatches is $H(x)$ for the service zone at $x \in \mathbf{R}$. The dispatched TUs first run on the line-haul segment toward the suburb (called inbound trips). And then, they *decouple* into individual MAVs to drop off and pick up patrons following a well-planned traveling-salesman-problem (TSP) tour within service zones. On the return trips to the terminal (called outbound trips), the MAVs may or may not be *re-coupled* into TUs depending on the optimized design for each service zone. If re-coupling is required, they must wait for the last MAV to finish the drop-offs and pick-ups; otherwise, they return separately to the terminal after completing their respective tasks in the zone.

The service design of the above-described MAV-based feeder system concerns four decision functions regarding location $x \in \mathbf{R}$. They are the service zone size $A(x)$ [km^2/zone], the service headway $H(x)$ [hour/dispatch], the transit unit (TU) size $S(x) \in \{c, 2c, ..., Kc\}$ expressed in unit of seats per dispatch, and the indicator $\delta(x) \in \{0, 1\}$ denoting the outbound MAVs are re-coupled in TUs ($\delta(x) = 1$) or not ($\delta(x) = 0$).

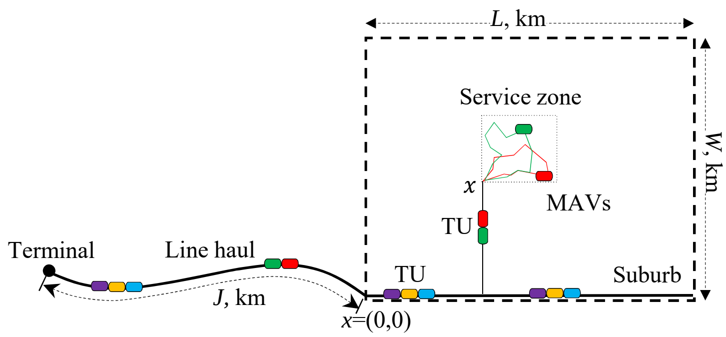

Figure 1 – *Layout of modular autonomous vehicle-based on-demand feeder service.*

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We construct the optimal design as a minimization problem of the generalized system cost, denoted Z, concerning $H(x), A(x), S(x), \delta(x), x \in \mathbf{R}$. The problem is formulated as below:

$$\underset{H(x),A(x),S(x),\delta(x)}{\text{minimize}} \; Z = \int_{x\in\mathbf{R}} \frac{1}{A(x)} \left[\delta(x) \left(\frac{1}{\mu} C_o(x) + C_p(x) \right) + (1 - \delta(x)) \left(\frac{1}{\mu} \tilde{C}_o(x) + \tilde{C}_p(x) \right) \right] dx, \tag{1a}$$

subject to:

$$\lambda_u(x) A(x) H(x) = S(x), \forall x \in \mathbf{R}, \tag{1b}$$

$$\frac{S(x)}{c} = I \in \{1, 2, ..., K\}, \forall x \in \mathbf{R}, \tag{1c}$$

$$H(x) \geq H_{\min}, A(x) \geq 0, \delta(x) \in \{0, 1\}, \forall x \in \mathbf{R}, \tag{1d}$$

where the integrand in Eq. (1a) yields the local generalized system cost per square km, and μ is patrons' average value of time that is used to convert monetary cost into the unit of time.

The $C_o(x), C_p(x)$ in Eq. (1a) are transit agency cost and patrons' trip time, respectively, under services that re-couple MAVs into TUs on the return trips from suburban zone at x to the terminal; and $\tilde{C}_o(x), \tilde{C}_p(x)$ correspond to the cost components without return-trip re-coupling.

The agency costs $C_o(x), \tilde{C}_o(x)$ are computed as follows:

$$C_o(x) = (\alpha + \beta S(x)) \frac{2D(x)}{H(x)V} + \gamma \frac{S(x)}{H(x)c} \mathrm{E} \left(\max_{i=1,2,...,I} \{t_i(x)\} \right), \tag{2a}$$

$$\tilde{C}_o(x) = (\alpha + \beta S(x)) \frac{D(x)}{H(x)V} + \gamma \left[\frac{S(x)}{c} \frac{D(x)}{H(x)V} + \frac{S(x)}{H(x)c} \mathrm{E}\left(t_i(x)\right) \right], \tag{2b}$$

where the first and second right-hand-side (RHS) terms return operation costs related to vehicle hours in TUs and MAVs, respectively. The $\alpha + \beta S(x)$ is the unit operation cost of TUs, which is determined by a fixed cost, α, and a variable energy cost, β, that is associated with capacity, $S(x)$. The γ is the unit operation cost of individual MAVs. The $D(x) := J + ||x||$ is the non-stop distance between the service zone at x and the terminal. The $t_i(x)$ denotes the i^{th} MAV's TSP-tour time in the service zone at x. Let σ be the standard deviation of $t_i(x)$, we have

$$\mathrm{E}\left(\max_{i=1,2,\dots,I}\{t_i(x)\}\right) = \mathrm{E}\left(t_i(x)\right)\left[1+\tau\left(x,S(x)\right)\right], \tag{3}$$

where $\tau(x,S(x)) := \frac{\sqrt{3}\sigma(S(x)-c)}{S(x)+c}$ is the ratio between additional time for re-coupling and the expected TSP time; see the full-length paper for deviation. The $\mathrm{E}(t_i(x))$ can be estimated by

$$\mathrm{E}(t_i(x)) = \frac{k}{\overline{V}}\sqrt{A(x)o_v(x)} + \frac{k}{\overline{V}}\sqrt{\frac{A(x)o_u(x)}{I}}, \tag{4}$$

where the first RHS term yields the expected TSP-tour time of an individual MAV for drop-offs and the second for pickups; see again full-length paper. The k is a dimensionless constant; the $\overline{V}$ is the vehicle's commercial speed discounted from the cruising speed V to account for delays in drop-offs and pick-ups; and $o_v(x)$ and $o_u(x)$ are the vehicle occupancy of inbound and outbound trips for service zone at x, respectively. They are $o_u(x) = c, o_v(x) = \frac{c\lambda_v(x)}{\lambda_u(x)}, \forall x \in \mathbf{R}$.

Ptrons' trip time for outbound demand, $\lambda_u(x)$, and inbound demand, $\lambda_v(x)$, consists of three components: (i) waiting time for the service to begin, $\frac{H(x)}{2}$; (ii) non-stop traveling time to/from the terminal, $\frac{D(x)}{V}$; and (iii) TSP-tour waiting time. Therefore, we have:

$$C_p(x) = \lambda_u(x)A(x)\left(\frac{H(x)}{2}+\frac{D(x)}{V}+(1+\tau(x,S(x)))\frac{kc}{\overline{V}}\sqrt{\frac{A(x)}{S(x)}}\right)+\lambda_v(x)A(x)\left(\frac{H(x)}{2}+\frac{D(x)}{V}+\frac{k}{2\overline{V}}\sqrt{\frac{A(x)c\lambda_v(x)}{\lambda_u(x)}}\right), \tag{5a}$$

$$\tilde{C}_p(x) = \lambda_u(x)A(x)\left(\frac{H(x)}{2}+\frac{D(x)}{V}+\frac{kc}{\overline{V}}\sqrt{\frac{A(x)}{S(x)}}\right)+\lambda_v(x)A(x)\left(\frac{H(x)}{2}+\frac{D(x)}{V}+\frac{k}{2\overline{V}}\sqrt{\frac{A(x)c\lambda_v(x)}{\lambda_u(x)}}\right). \tag{5b}$$

Problem (1) is equivalent to the following problem according to the calculus of variations.

$$\underset{H(x),A(x),S(x),\delta(x)}{\text{minimize}} \quad z(x,H(x),A(x),S(x),\delta(x)) \text{ for each } x\in\mathbf{R}\text{, subject to Eqs. (1b–1d).} \tag{6}$$

We take $\delta(x) = 1$ for illustration and assume for now $S(x) = S^*(x)$, where $S^*(x)$ is the optimal value of TU size. By constraint (1b), we can rewrite the objective function in (1a) as,

$$z(x,H(x)|S^*(x),\delta(x)=1) = z(x,H(x),S(x)=S^*(x),\delta(x)=1), \tag{7}$$

Eq. (7) is a posynomial function regarding $H(x)$, yielding unique global optima in optimization.

$$\widehat{H}(x|S^*(x),\delta(x)=1) = \left(\frac{1+\tau(x,S^*(x))}{\lambda_u(x)+\lambda_v(x)}\frac{k}{\overline{V}}\left(\frac{\gamma}{\mu}\left(\sqrt{\lambda_u(x)}+\sqrt{\frac{S^*(x)\lambda_v(x)}{c}}\right)+c\sqrt{\lambda_u(x)}+\frac{\lambda_v(x)}{2\lambda_u(x)}\frac{\sqrt{cS^*(x)\lambda_v(x)}}{1+\tau(x,S^*(x))}\right)\right)^{\frac{2}{3}}. \tag{8}$$

Combing the boundary condition (1d) obtains the optimal headway,

$$H^*(x|S^*(x),\delta(x)=1) = \max\left\{\widehat{H}(x|S^*(x),\delta(x)=1), H_{\min}\right\}, \tag{9}$$

and accordingly the optimal zone size,

$$A^*(x|S^*(x),\delta(x)=1) = \frac{S^*(x)}{\lambda_u(x)H^*(x|S^*(x),\delta(x)=1)}. \tag{10}$$

Substituting (9) and (10) back to the objective function produces the local system cost function $z(x,S^*(x)|\delta(x)=1)$ with a single unknown $S^*(x)$, which can be found efficiently via enumeration, i.e.,

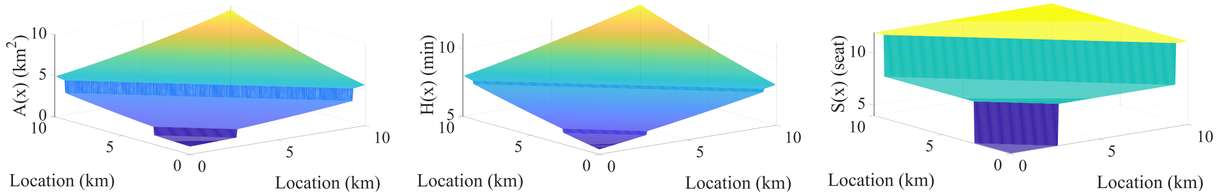
$$S^*(x|\delta(x)=1) = \mathrm{argmin}_{S^*(x)\in\{c,2c,\dots,Kc\}}\ z(x,S^*(x)|\delta(x)=1). \tag{11}$$

Similarly, we can find the solution under the condition of $\delta(x) = 0$.

3 Results and discussions

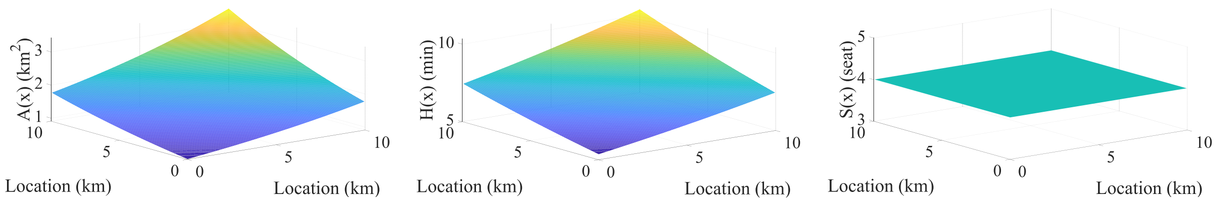
We use a distance-decaying demand with $\lambda_u(x) = \overline{\lambda}_u \exp(-\eta_u(||x||)), \lambda_v(x) = \overline{\lambda}_v \exp(-\eta_v(||x||))$, where $\overline{\lambda}_u = 50, \overline{\lambda}_v = 10$ [trips/km^2/h], $\eta_u, \eta_v \in \{0, 0.1, 0.5\}$ representing uniform, less-heterogeneous and more-heterogeneous demands. The baseline parameter values are omitted here for brevity.

Figures 2-3 visualize heterogeneous designs of MAV- and bus-based ODFT. As seen, the optimized MAV-based ODFTs entail larger zone sizes and headways than bus-based systems. The TU sizes of MAVs increase with distance.



(a) *Optimized zone size $A^*(x)$.* (b) *Optimized headway $H^*(x)$.* (c) *Optimized TU size $S^*(x)$.*

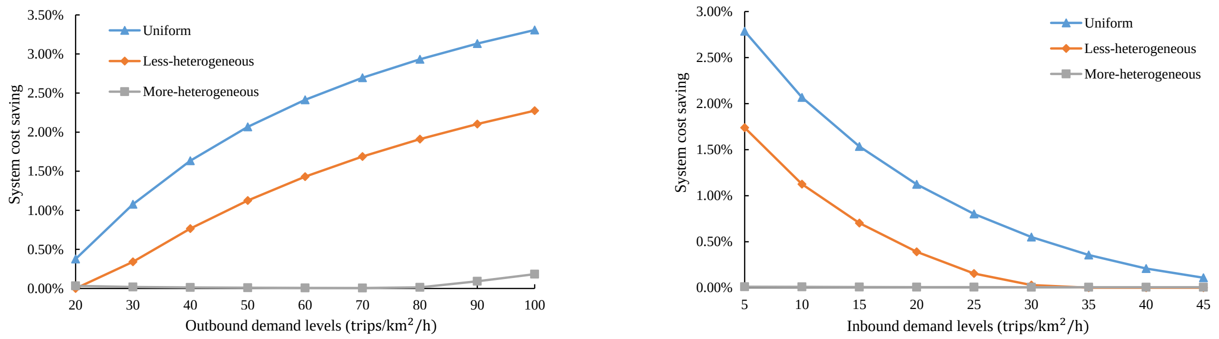
Figure 2 – *Optimal designs for MAV-based ODFT ($\eta_u = \eta_v = 0.1$).*



(a) *Optimized zone size $A^*(x)$.* (b) *Optimized headway $H^*(x)$.* (c) *Optimized TU size $S^*(x)$.*

Figure 3 – *Optimal designs for bus-based ODFT ($\eta_u = \eta_v = 0.1$).*

Figure 4 presents MAV-based ODFTs' cost savings against bus-based systems in different demand scenarios. We observe that the cost savings increase with the growth of outbound demand, but decrease with the rising inbound demand. This is because each module of inbound and outbound MAVs traverses across areas of $A(x)$ and $A(x)/I$, respectively, meaning that only outbound demand enjoys benefit from the decoupling of MAVs.



(a) *Cost saving for different outbound demand levels.* (b) *Cost saving for different inbound demand levels .*

Figure 4 – *Cost savings against bus-based ODFT ($\eta_u = \eta_v = 0.1$).*