An end-to-end predict-then-optimize method for on-demand vehicle relocation in mobile sensing

X. Wang [a] and W. Ma [a*]

[a] Civil and Environmental Engineering, The Hong Kong Polytechnic University, Hong Kong Special Administrative Region, China
xinyuu.wang@connect.polyu.hk, wei.w.ma@polyu.edu.hk
[*] Corresponding author

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1 INTRODUCTION

In recent years, ubiquitous mobile devices have rapidly promoted the development of mobile crowd sensing (MCS). As a specialization of MCS, vehicle crowd sensing (VCS) harnesses the sensing capabilities of the built-in sensors in vehicles to collect and analyze data (Xu *et al.*, 2019, Ji *et al.*, 2023). The VCS system comprises a crowdsourcer, a monitoring center, and vehicles. There are two types of vehicles: the controlled vehicles guided by monitoring centers and the free vehicles guided by drivers. Once the monitoring center receives the target sensing distribution from the crowdsourcer, the controlled vehicles are immediately relocated to their destinations. In the meantime, the free vehicles can select their preferred cruising areas. Then the question arises: how to accurately predict the movements of free vehicles and how to allocate controlled vehicles to achieve the desired target sensing distribution?

A conventional approach for the question is the two-stage predict-then-optimize method (PTO) which separates the prediction and optimization into 2 stages. However, training the prediction model based on prediction error can lead to inferior decision-making than directly minimizing the decision error (Elmachtoub & Grigas, 2022). To this end, we propose an end-to-end predict-then-optimize framework by integrating optimization into prediction. This framework requires computing the gradients of the optimization layer within the deep learning architecture. In general, there are two methods related to the problem: differentiating the optimality condition implicitly (Agrawal *et al.*, 2019, Amos & Kolter, 2017) and applying the unrolling methods explicitly(Sun *et al.*, 2023). The implicit method obtains the gradient directly by differentiating the Karush-Kuhn-Tucker (KKT) conditions but requires expensive computation. The explicit unrolling method computes the gradients iteratively which may accelerate the computational speed and solve large-scale problems, but has not been well explored.

To this end, we develop a novel end-to-end smart predict-then-optimize (SPO) framework embedded with an unrolling differentiation method for the real-time prediction and vehicle relocation problem to achieve a target sensing distribution. This paper has three major contributions:

1. *A novel end-to-end smart predict-and-optimize framework.* We construct an end-to-end framework based on SPO and integrate a quadratic relocation problem as the differentiable

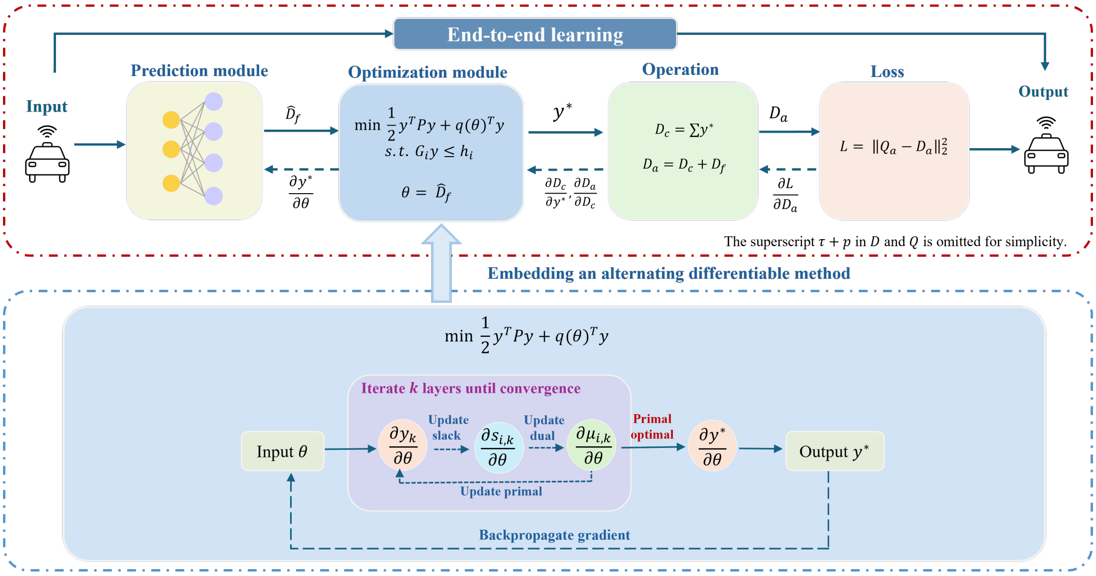

Figure 1 – *The end-to-end SPO framework for vehicle prediction and relocation problem*

layer in the neural network. This end-to-end framework is trained by minimizing the task-specific matching error and allows us to obtain the actual optimal strategy.

2. *A comparable unrolling algorithm to differentiate the optimization layer.* We apply an unrolling method based on ADMM to compute the gradients of the optimization layer, which can balance the trade-off between the solution accuracy and the computation efficiency.

3. *An effective solving approach for large-scale networks.* The alternating differentiation method allows us to solve the large-scale network problem within an acceptable time.

2 METHODOLOGY

The overall end-to-end SPO framework for vehicle prediction and relocation is shown in Figure 1. The framework consists of two essential modules: a prediction module and an optimization module. The prediction module is designed based on the spatial-temporal neural network to predict the short-term free vehicle demand. The optimization module formulates the relocating problem for controlled vehicles with an alternating differentiation method adapted from ADMM to generate the optimal relocation strategy. The ultimate goal is to make the matching distribution combined by free vehicles and controlled vehicles close to the target sensing distribution.

2.1 Model

2.1.1 Prediction model

For each time interval τ, denote the spatial distribution of free vehicles, controlled vehicles, and all vehicles as D_f^τ, D_c^τ, D_a^τ respectively, denote the target sensing distribution at time τ as Q_a^τ. The demand for free vehicles $\hat{D}_f^{\tau+p}$ at time $\tau + p$ is predicted by a two-layer TGCN model with M lookback windows. Then the target distribution of controlled vehicles can be easily obtained by $\hat{D}_c^{\tau+p} = Q_a^{\tau+p} - \hat{D}_f^{\tau+p}$, which will be used in the relocation stage.

2.1.2 Vehicle relocation problem

We aim to satisfy the matching distribution of controlled vehicles $\hat{D}_c^{\tau+p}$ in the relocation stage, by considering the total travel time budget (System Optimal), accessibility constraint (all vehicles can arrive at each time interval), and supply limitation. Denote $x_{c,ij}^{\tau+p}$ as the controlled vehicle flow from grid i to grid j at time $\tau+p$. Then the relocation problem is formulated in Eq. (1):

$$
\begin{aligned}
\underset{x_{c,ij}^{\tau+p}}{\text{minimize}} \quad & Z_0 = \frac{1}{2}\sum_j \|\sum_i x_{c,ij}^{\tau+p} - \hat{D}_{c,j}^{\tau+p}\|_2^2 && \text{(1a)}\\
\text{subject to} \quad & \sum_i x_{c,ij}^{\tau+p} \leq D_{c,i}^{\tau}, \forall i \in I, j \in J, \tau \in T, && \text{(1b)}\\
& (c_{ij}^{\tau} - \tau) \odot x_{c,ij}^{\tau+p} \leq 0, \forall i \in I, j \in J, \tau \in T, && \text{(1c)}\\
& \sum_i \sum_j c_{ij}^{\tau} x_{c,ij}^{\tau+p} \leq K, \forall i \in I, j \in J, \tau \in T, && \text{(1d)}\\
& -x_{c,ij}^{\tau+p} \leq 0, \forall i \in I, j \in J, \tau \in T && \text{(1e)}
\end{aligned}
$$

where c_{ij}^{τ} is the travel time from grid i to j at time τ; K is the maximum total travel time; $\odot$ is the Hadamard product. The variables in the original relocation formulation are then vectorized and flattened in one dimension and therefore $x_{ij} \in \mathbb{R}^{N\times N}$ is converted to $\mathbf{y} \in \mathbb{R}^{N^2}$, The output of prediction module $\hat{D}_f^{\tau+p}$ is replaced by θ for simplicity. The new objective function is $Z_1 = \frac{1}{2}\mathbf{y}^T\mathbf{P}\mathbf{y} + \mathbf{q}(\theta)^T\mathbf{y}$, and the four constraints are generalized by: $\mathbf{G_i}\mathbf{y} \leq \mathbf{h_i}, \forall i \in \{1,2,3,4\}$.

2.1.3 Integrated model

From the relocation stage, We obtain the actual distribution of controlled vehicles $D_c^{\tau+p}$ by aggregating $\mathbf{y}$. Note that at time $\tau+p$, all free vehicles have already arrived at their destinations so the actual distribution of free vehicles $D_f^{\tau+p}$ is easily obtained. Therefore, the actual matching distribution for all vehicles at time $\tau+p$ is represented by $D_a^{\tau+p} = D_c^{\tau+p} + D_f^{\tau+p}$. The loss function L for the SPO framework is formulated by minimizing the matching error between $D_a^{\tau+p}$ and $Q_a^{\tau+p}$. Detailed formulations will be presented in the full paper.

2.2 Solution approach

Inspired by the alternating differentiation framework proposed by (Sun *et al.*, 2023), we develop an unrolling differentiation method in the optimization layer to obtain the derivatives of primal variables with respect to the pre-defined parameters θ. We first decouple the constrained quadratic problem into sub-problems based on ADMM, then update the differentiation of the primal, slack, and dual variables y, s_i, μ_i alternatively. Following the procedures, We obtain the explicit differentiation function as summarized in Eq. (2) and Eq. (3), detailed derivations will be presented in the full paper:

$$
\begin{cases}
y_{k+1} = -(P + \sum_{i=1}^{4}\rho G_i^T G_i)^{-1}(q(\theta) + \sum_{i=1}^{4}\rho G_i^T(s_i - h_i) + \sum_{i=1}^{4} G_i^T \mu_i) & \text{(2a)}\\
s_{i,k+1} = \mathbf{ReLU}(-\frac{1}{\rho}\mu_{i,k} - (G_i y_{k+1} - h_i)) \quad \forall i \in \{1,2,3,4\} & \text{(2b)}\\
\mu_{i,k+1} = \mu_{i,k} + \rho(G_i y_{k+1} + s_{i,k+1} - h_i) \quad \forall i \in \{1,2,3,4\} & \text{(2c)}
\end{cases}
$$

$$
\begin{cases}
\frac{\partial y_{k+1}}{\partial \theta} = -(P + \sum_{i=1}^{4}\rho G_i^T G_i)^{-1}(\frac{\partial q(\theta)}{\partial \theta} + \sum_{i=1}^{4}\rho G_i^T \frac{\partial s_i}{\partial \theta} + \sum_{i=1}^{4} G_i^T \frac{\partial \mu_i}{\partial \theta}) & \text{(3a)}\\
\frac{\partial s_{i,k+1}}{\partial \theta} = -\frac{1}{\rho}\mathbf{sgn}(s_{i,k+1}) \cdot \mathbf{1}^T \odot (\frac{\partial \mu_{i,k}}{\partial \theta} + \rho\frac{\partial (G y_{i,k+1} - h_i)}{\partial \theta}) \quad \forall i \in \{1,2,3,4\} & \text{(3b)}\\
\frac{\partial \mu_{i,k+1}}{\partial \theta} = \frac{\partial \mu_{i,k}}{\partial \theta} + \rho\frac{\partial (G y_{i,k+1} + s_{i,k+1} - h_i)}{\partial \theta} \quad \forall i \in \{1,2,3,4\} & \text{(3c)}
\end{cases}
$$

Table 1 – *Performance of SPO-A and other baselines in HK dataset*

Grid size	Total budget	**SPO-A(Ours)**		SPO-C		PTO		DON	
		RMSE	MAPE(%)	RMSE	MAPE(%)	RMSE	MAPE(%)	RMSE	MAPE(%)
45	8000	**9.072**	31.623	9.510	**31.551**	9.563	34.081	14.518	49.655
	10000	9.647	31.552	**9.541**	**31.457**	9.551	33.987	14.518	49.655
	15000	**9.059**	**31.426**	9.507	31.431	9.563	34.087	14.518	49.655
68	8000	**9.834**	**33.479**	-	-	10.701	38.166	15.449	52.643
	10000	**9.689**	**32.966**	-	-	10.519	36.955	15.449	52.643
	15000	**9.752**	**33.277**	-	-	10.547	37.495	15.449	52.643

3 RESULTS

We evaluate the proposed end-to-end SPO framework on the taxi dataset in the Kowloon District, Hong Kong SAR. The study area is discretized into hexagon grids, and taxi data was collected from HKTaxi from March 14 to March 22, 2023. We aggregate the taxi demand in each grid every 15 minutes and split the taxi dataset with a ratio of 7:1:1 for training, validation, and testing. We select 45 and 68 grids to represent mid-size and large-scale networks. The ratio of controlled vehicles to free vehicles is 6 : 4. We compare the proposed SPO model with the alternating method (SPO-A) with three baseline methods: SPO with an implicit method CVXPY(Agrawal *et al.*, 2019) (SPO-C), PTO, and the do-nothing method(DON) as presented in Table 1. Results show that the proposed SPO framework outperforms the benchmarks in large-scale networks, though slightly under-performs in mid-size networks.

4 DISCUSSION

In this paper, we propose a novel SPO framework for real-time vehicle relocation to realize a target sensing distribution in vehicle crowd sensing. We embed a quadratic relocation problem into a neural network and develop an alternating differentiation approach for fast and recursive solutions. Results show that the proposed framework with the unrolling differentiation method shows comparable matching performance in mid and large-scale networks. In future work, we will conduct ablation studies and sensitivity analysis by changing different prediction modules, control ratios, and target distributions to evaluate the robustness of the proposed SPO framework.

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