

# Distributed Adaptive Signal Control based on Shockwave Theory

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## 1 INTRODUCTION

Advanced traffic control methods with considerable flexibility possess great potential to improve urban traffic efficiency and stability. This study introduces two distributed adaptive control methods based on a novel evaluation index, termed '*synthetic delay*', accommodating to various traffic conditions. The first method optimizes phase sequence in real-time, while the second method coordinates green time and traffic advisory speed with fixed phase sequence.

Adaptive control methods optimize traffic signal schemes in real-time with diverse objectives under different conditions, such as minimizing total delay, maximizing throughput, etc. (Hajbabaie & Benekohal, 2013, Peng & Wang, 2023). While effective in specific scenarios, these methods face challenges in providing uniform control across constantly changing traffic conditions. Additionally, the complexity of centralized control limits their applicability in large-scale traffic systems, prompting the need for distributed control methods that offer simpler computation and greater flexibility (Jiang *et al.*, 2021, Varaiya, 2013).

This study introduces a novel evaluation index for traffic signal control accommodating to various traffic conditions, and presents two distributed adaptive control methods. Leveraging shockwave theory, traffic dynamics at intersections are captured by integral in the index referred to as '*synthetic delay*', which automatically evaluates both delay and throughput with flexible significance. Taking the synthetic delay as optimization objective, the first control method selects optimal phases in real-time, while the second method coordinates green time and advisory traffic speed based on fixed phase sequence. Numerical experiments conducted in a grid network demonstrate that the proposed methods significantly reduce average delay and increase network throughput under different conditions, thereby enhancing traffic efficiency.

## 2 METHODOLOGY

A novel evaluation index termed '*synthetic delay*' is introduced, upon which two distributed adaptive control methods established. Section 2.1 explains the synthetic delay, and its adaptability to diverse conditions. In Section 2.2, the distributed adaptive control method for dynamically optimizing phase sequences is presented, followed by the distributed adaptive control method coordinates green time and advisory speed for arrival traffic described in Section 2.3.

## 2.1 Synthetic delay adapting to various conditions

The synthetic delay is established through integral calculations based on shockwave theory, considering queue formation and dissipation at intersections and flow fluctuations resulting from phase switches at upstream intersections. It automatically evaluates delay and throughput with adaptable significance under varied conditions, providing assessments of signal control efficiency.

Shockwave theory is applied to depict traffic dynamics at intersections as illustrated in Fig.1. During red time, the traffic is blocked after initial queues, manifested by shockwave  $y_1$ . As green phases commence, queues begin to dissipate, forming a subsequent shockwave  $y_2$ . Besides, phase switches at upstream intersections contribute to traffic flow variations and influence the queue formation at downstream intersections, depicted by shockwaves  $y_3$  and  $y_4$  respectively. The formation and dissipation of queue at the intersection are represented by yellow polygon in Fig.1, while the green polygon is considered for the flow changes caused by phase switch at upstream intersection. The synthetic delay is formulated by applying integrals to these components:

$$\begin{aligned}\Pi &= \int_{x_A}^{x_B} (L - y_1)dx + \int_{x_B}^{x_C} (y_2 - y_1)dx - \left( \int_{x_D}^{x_C} (y_2 - y_1)dx - \int_{x_D}^{x_E} (y_2 - y_4)dx \right) \\ &= \int_{x_A}^{x_B} Ldx - \int_{x_A}^{x_D} y_1dx + \int_{x_B}^{x_E} y_2dx - \int_{x_D}^{x_E} y_4dx\end{aligned}$$

where  $\Pi$  is the synthetic delay,  $x$  is the abscissa of corresponding point in Fig.1,  $L$  is the distance between two intersections.

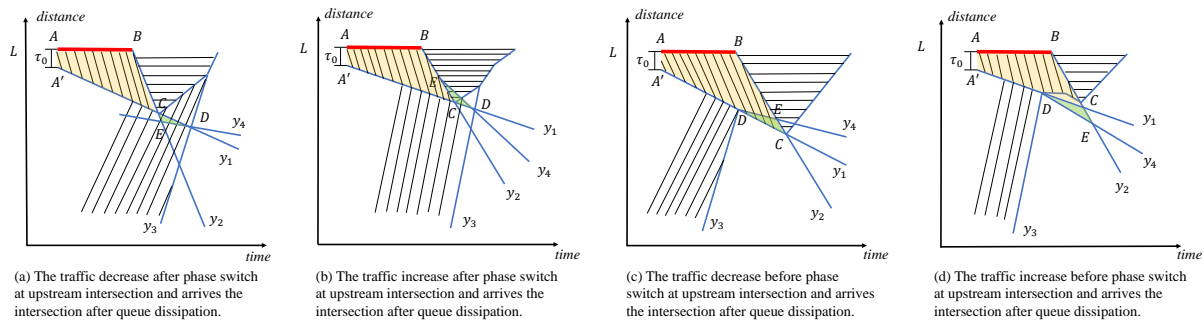


Figure 1 – Traffic dynamics at intersections, where  $\tau_0$  is the initial queue length,  $L$  is the distance between intersections.

Compared to traditional optimization objectives considering simple measurements, the synthetic delay automatically evaluates delay and throughput with flexible importance. The integral  $\int_{x_A}^{x_B} (L - y_1)dx + \int_{x_B}^{x_C} (y_2 - y_1)dx$  represented by yellow polygon contributes to total delay without flow fluctuations. Considering the influence of phase switch at upstream intersection on traffic flow, the integral  $\int_{x_D}^{x_C} (y_2 - y_1)dx - \int_{x_D}^{x_E} (y_2 - y_4)dx$  related to green polygon modifies the delay and evaluates throughput as well: a traffic flow increase results in a positive value, suggesting more traffic is blocked during red time after phase switch; while a negative value deriving from flow decrease indicates mitigation of queue. Furthermore, the synthetic delay adapts to different saturations, diverse flow fluctuations, and various fundamental diagrams by variations of shockwave slopes and locations.

## 2.2 Optimization of signal control for phase sequence

Building upon the synthetic delay, a signal control method selecting the optimal phases at each time step is designed with more flexibility and adaptation. Additionally, the queue length at downstream intersection and availability for minor street traffic are taken into account.

This method aims to mitigate the impact caused by signal control by activating the optimal phase from a predefined finite set of feasible phases  $\Phi_j$  at intersection  $j$  at each control step. The

selection of the optimal phase is guided by a compound index for the entire intersection, which encompasses synthetic delay, queue penalty and interval penalty as the selection criteria:

$$\gamma_i = \alpha \Pi_i / \Pi_{max} - (1 - \alpha) p_{q,i} + p_{m,i} \quad (1)$$

$$p_{q,i} = \frac{l_{0,j+1,i}}{c_i} \quad (2)$$

$$p_{m,i} = \exp(m_i - m_0) \frac{l_{0,j,i}}{\bar{l}_{0,j,i}} \quad (3)$$

where  $\gamma_i$  is compound index of flow  $i$ ,  $p_{q,i}$  is queue penalty of flow  $i$ ,  $p_{m,i}$  is interval penalty of flow  $i$ ,  $\Pi_{max}$  is the maximum synthetic delay of all flows,  $\alpha$  is weight coefficient from 0 to 1.  $l_{0,j,i}$  and  $\bar{l}_{0,j,i}$  are current queue length and history queue length of flow  $i$  at intersection  $j$ ,  $c_i$  is the capacity of road for flow  $i$ ,  $m_i$  is waiting time of flow  $i$ , and  $m_0$  is the threshold of waiting time.

The optimal phase is selected to activate at every time step to improve the traffic efficiency at intersections. The stability of the method can be proven based on Markov chain theory.

### 2.3 Distributed adaptive control for green time and advisory speed

Given the fixed phase sequence, the synthetic delay is utilized to optimize the green time for the next phase by differential. Furthermore, treating the synthetic delay as a multivariable function of green time and arrival traffic speed, the signal control and traffic advisory speed are coordinated to reduce stoppage at intersections and improve the traffic efficiency.

For the subsequent phase  $q$ , the optimization objective is to minimize the synthetic delay of the entire intersection. The total synthetic delay of the intersection is a quadratic function of the green time for phase  $q$  and reaches its unique global minimum value at the pole point. Therefore, the optimal green time is calculated by differential.

Furthermore, recognizing the synthetic delay as a multivariable function of green time and arrival traffic speed, the signal control and advisory speed are coordinated. The minimum value of synthetic delay is achieved through partial differential by solving the equation set:

$$\begin{cases} \frac{\Pi \partial (g_q, v_{1,opt}, v_{2,opt}, \dots, v_{|I|,opt})}{\partial g_q} = 0 \\ \frac{\Pi \partial (g_q, v_{1,opt}, v_{2,opt}, \dots, v_{|I|,opt})}{\partial v_{i,opt}} = 0, \quad i = 1, 2, \dots, |I| \end{cases} \quad (4)$$

where  $\Pi$  is the total synthetic delay,  $g_q$  is the optimal green time of phase  $q$ ,  $I$  is set of all flows at the intersection with element number of  $|I|$ ,  $v_{i,opt}$  is the optimal advisory speed for flow  $i$ .

Since the practical complexity of the equation set, genetic algorithm is applied to solve the problem for its simplicity and convenience.

## 3 RESULTS AND DISCUSSION

The proposed control methods are evaluated in a 5\*5 grid network and compared to Max-pressure method (Varaiya, 2013) and traditional adaptive control method by SUMO, with the traffic demands ranging from 50 veh/h to 2500 veh/h. The average delay, cumulative traffic of the network, and average number of stoppages are measured for assessment. Results demonstrate the superiority of the proposed methods across various traffic conditions. Average delay reduction ranged from 2.90% to 82.10%, with a cumulative flow increase exceeding 20% under saturated and congested traffic. Moreover, the signal control method optimizing phase sequence reduces the average delay significantly, while the adaptive control method coordinating green time and advisory speed decreases the number of stoppages at intersections. Additionally, the proposed methods promote the green wave for unsaturated traffic, and lead to homogeneous speed distribution under congested conditions.

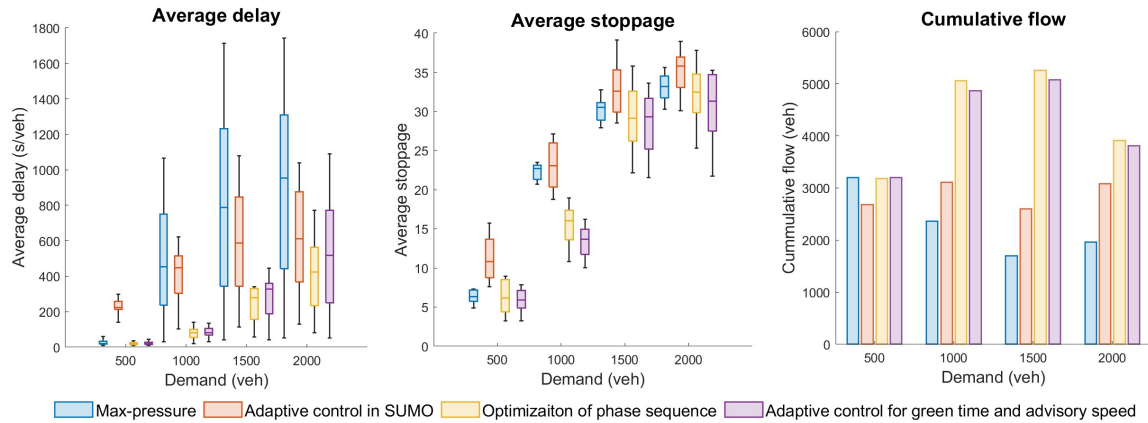


Figure 2 – Results in the network

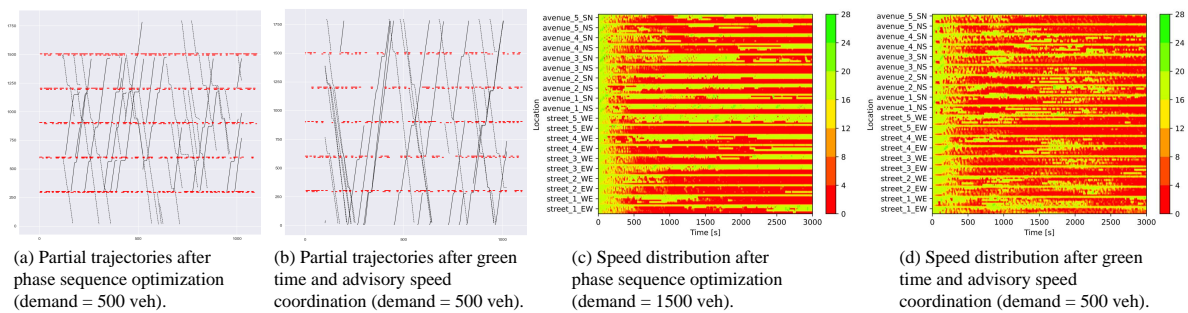


Figure 3 – Partial trajectories and speed distributions in the network

## 4 CONCLUSION

This study proposes two distributed adaptive signal control methods based on a novel evaluation index, with flexibility in adapting to various traffic conditions. Leveraging shockwave theory, an index termed '*synthetic delay*' is introduced, which depicts traffic dynamics at intersections by integral. It automatically evaluates delay and throughput with adaptable significance, and accommodates to different saturations, diverse flow fluctuations, and various fundamental diagrams. Applying the synthetic delay as optimization objective, two distributed adaptive control methods are presented to optimize the phase sequence and green time respectively. Numerical experiments demonstrate the effectiveness of proposed methods in reducing delay and increasing throughput across diverse traffic conditions. Nonetheless, future research should address flow fluctuations, possibly through advanced traffic prediction techniques.

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