Dynamic system optimal pricing for shared autonomous vehicles in congestible networks: Theoretical properties

Toru Seo[a,*], Ryota Maruyama[a], Kentaro Wada[b] and Yikai Zhou[b]

[a] Tokyo Institute of Technology, Tokyo, Japan
seo.t.aa@m.titech.ac.jp

[b] University of Tsukuba, Ibaraki, Japan
wadaken@sk.tsukuba.ac.jp

[*] Corresponding author

Extended abstract submitted for presentation at the Conference in Emerging Technologies in Transportation Systems (TRC-30)
September 02-04, 2024, Crete, Greece

April 15, 2024

Keywords: SAV system, dynamic traffic assignment, congestion pricing, two-sided market

1 INTRODUCTION

Shared autonomous vehicle (SAV) systems could be a promising transportation mode in the near future (Narayanan *et al.*, 2020). In an SAV system, large number of autonomous vehicles are shared by the society, and they transport people with ridesharing in an optimized manner. Therefore, it is expected that SAV systems will be more efficient than the current privately owned vehicles and more flexible than the current fixed-route public transport. In the literature, the optimization of SAV systems, such as finding optimal routing and ridesharing matching, has been extensively studied (e.g., Levin, 2017).

How to realize system optimal (SO) states of SAV systems in which travelers and SAV operators seek their own profit, is also important. To answer this question, number of researches have analyzed market economics or static traffic equilibrium of SAV systems (or related shared mobility services), proposed promising approaches such as optimal pricing, and obtained several policy implications (e.g., Ke *et al.*, 2021, Ke & Qian, 2023, Kashmiri & Lo, 2024).

The dynamic natures of SAV systems is also important. For example, temporal demand concentration may cause long waiting time for passengers, and the system administrator have to take measure for it by charging surge pricing for passengers or providing incentives to SAV operators in order to improve system's performance (just like the current ridesourcing systems (Yang *et al.*, 2020)). In order to find the optimal solutions for this kind of measures, dynamic analysis of SAV systems are necessary. However, to the authors' knowledge, mathematically tractable analysis on this problem is very limited. Existing studies on dynamic operation management of SAV systems employ complicated methodologies such as deep reinforcement learning (Xie *et al.*, 2023), Bayesian optimization (Liu *et al.*, 2024), non-equilibrium models (Ramezani & Valadkhani, 2023). They are very useful to find the optimal solution for specific cases, but they may not be convenient to find general theoretical implications.

In this study, we mathematically analyze dynamic operation of an SAV system and derive several properties on the optimal pricing for it. Specifically, we develop a model of SAV systems where the behaviors of travelers and SAV operators follow the *dynamic user equilibrium (DUE)* principle in a congestible many-to-many network. By analyzing the model, we mathematically derive *dynamic system optimal (DSO)* pricing for the SAV system. Then, we prove several theorem on the optimal pricing and SAV system. Some of the findings can be summarized as follows.

- In the DSO state, travelers may pay congestion toll to SAV operators, and SAV operators may pay another congestion toll to the road authority.
- The congestion toll is charged when and where the road and/or SAV seat is congested.
- The SAV operation cost including the road toll is fully covered by toll from passengers.
- The optimal SAV fleet size is automatically maintained by toll from passengers.

2 METHODOLOGY

The overview of our methodology is as follows and shown in Fig. 1. First, we develop a DSO model for SAV systems by modifying Seo & Asakura (2022). Then, we derive the dual problem of the DSO problem. We mathematically prove that the dual problem can be interpreted as a DUE model where the optimal pricing are charged to passengers and SAVs.

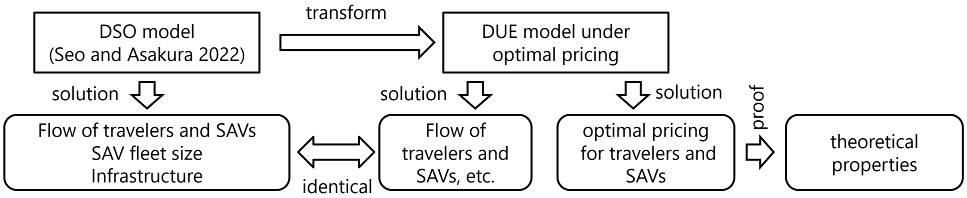


Figure 1 – *Modeling framework*

2.1 Dynamic system optimal model

The key assumptions of our DSO model is as follows:

- All travelers use an SAV system.
- Travelers' time-dependent OD demand is given.
- Travelers' cost is the weighted sum of waiting time, in-vehicle travel time, and scheduling cost.
- Each SAV travels through the road network and pickup/drop-off passengers appropriately.
- Each SAV has passenger capacity (e.g., they can transport 2 passengers simultaneously).
- Each road has traffic capacity.
- Each node has vehicle storage capacity that can be interpreted as queueing capacity or parking capacity.
- SAVs' cost are the weighted sum of travel distance and fixed maintenance cost.
- The number of SAV is a variable (i.e., we also consider fleet-sizing problem).
- Road and node capacities can be variables (i.e., we can also consider infrastructure planning problem).

With these assumptions, the underlying network traffic flow model is a *point-queue-based dynamic traffic assignment model with queue length constraints*. The model appropriately capture the important dynamic traffic phenomena of SAV systems, such as temporal and spatial traffic congestion, passengers' waiting behavior, empty SAV's routing to pickup passengers, and detour due to ridesharing (Seo & Asakura, 2022).

The model optimizes

- routing of SAVs and passengers,
- SAV fleet size, and
- infrastructure capacities,

to minimize the weighted sum of total travel time of travelers, total travel distance of SAVs, the SAV fleet maintenance cost, and infrastructure cost.

2.2 Dynamic user equilibrium under optimal pricing

By adopting the approach proposed by Akamatsu & Wada (2017), the aforementioned DSO model can be transformed as a DUE model under the optimal congestion pricing. In this DUE model, travelers and SAVs behave to minimize their own cost by taking *congestion pricing* charged by certain entities into account. It is proven that their cost satisfies the dynamical version of Wardrop's equilibrium condition. The dual variables corresponds to the capacity constraints in the original problem can be interpreted as a congestion pricing charged to the roads or SAVs.

3 THEORETICAL PROPERTIES

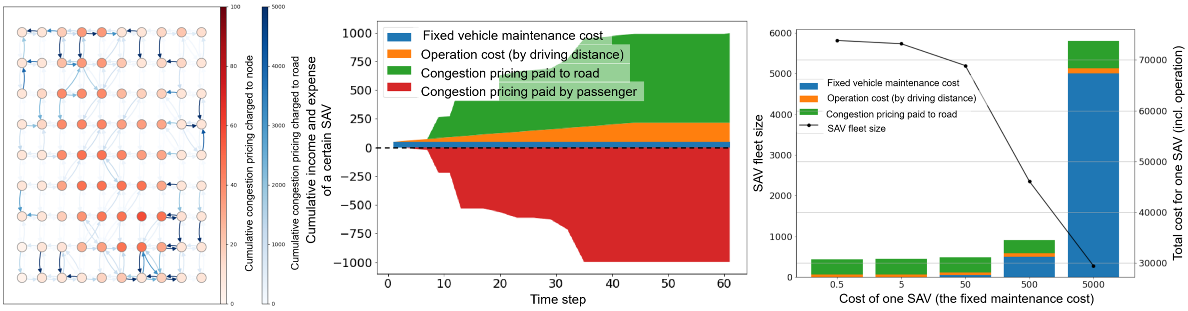
By analyzing the DUE model, we mathematically prove several theorems. For example, in a DUE state with the optimal congestion pricing, the following properties hold:

1. SAVs may charge congestion pricing to their passengers, and the road administrator may charge another one to SAVs. The amount varies on time and space.
2. If an SAV has vacant seat, no congestion pricing is charged to passengers who use the SAV at that time moment. Similarly, if a road is not congested, no congestion pricing is charged to SAVs that use the road at that time moment.
3. The infrastructure cost is always smaller than or equal to the sum of the congestion pricing paid by SAVs.
4. The cost of an SAV (i.e., operation cost, fixed maintenance cost, congestion pricing *paid to* the road admin) is always equal to the sum of the congestion pricing *paid by* its passengers.

The property 1 clarifies the flow of money in this two-sided market. The property 2 is qualitatively consistent to the marginal cost pricing principle. The properties 3 and 4 corresponds to the self-financing and revenue-neutral principles discussed in the conventional transportation (Nie & Liu, 2010). Among them, the property 4 would be the most significant result, as it implies that the optimal SAV fleet size can be automatically maintained by charging optimal pricing to the users.

4 NUMERICAL EXAMPLES

To show quantitative features, we have numerically solved the proposed model using actual travel data in a Japanese city with 9 km x 10 km area and 48000 travelers. Some results are illustrated in Fig. 2. In Fig. 2a, the optimal toll differs by location reflecting road and demand patterns. In Fig. 2b, the cost composition for a certain SAV is confirmed; note that the income and expense is completely balanced as mathematically proven. In Fig. 2c, sensitivity analysis on SAV's cost is shown; this kind of results might be useful to plan the future transportation system.



(a) Spatial distribution of cost charged for SAVs

(b) Dynamic cost composition of a certain SAV

(c) Sensitivity analysis on SAV cost

Figure 2 – *Numerical results*

5 CONCLUSION

Theoretical analysis results on SAV systems in dynamic congestible network are obtained. To the authors' knowledge, this kind of general results have not been obtained for SAV systems. The proposed methodology and the results would be useful to plan the future transportation systems. The future work includes incorporation of other transportation modes, extension to microscopic/disaggregate models, and analysis on cooperative SAVs (i.e., oligopoly market).

References

Akamatsu, Takashi, & Wada, Kentaro. 2017. Tradable network permits: A new scheme for the most efficient use of network capacity. *Transportation Research Part C: Emerging Technologies*, **79**, 178–195.

Kashmiri, Faizan Ahmad, & Lo, Hong K. 2024. Routing of multi-modal autonomous vehicles for system optimal flows and average travel cost equilibrium over time. *Transportation Research Part C: Emerging Technologies*, **159**, 104483.

Ke, Jintao, Li, Xinwei, Yang, Hai, & Yin, Yafeng. 2021. Pareto-efficient solutions and regulations of congested ride-sourcing markets with heterogeneous demand and supply. *Transportation Research Part E: Logistics and Transportation Review*, **154**, 102483.

Ke, Zemian, & Qian, Sean. 2023. Leveraging ride-hailing services for social good: Fleet optimal routing and system optimal pricing. *Transportation Research Part C: Emerging Technologies*, **155**, 104284.

Levin, Michael W. 2017. Congestion-aware system optimal route choice for shared autonomous vehicles. *Transportation Research Part C: Emerging Technologies*, **82**, 229–247.

Liu, Ao, Zhong, Shaopeng, Sun, Daniel, Gong, Yunhai, Fan, Meihan, & Song, Yan. 2024. Joint optimal pricing strategy of shared autonomous vehicles and road congestion pricing: A regional accessibility perspective. *Cities*, **146**, 104742.

Narayanan, Santhanakrishnan, Chaniotakis, Emmanouil, & Antoniou, Constantinos. 2020. Shared autonomous vehicle services: A comprehensive review. *Transportation Research Part C: Emerging Technologies*, **111**, 255–293.

Nie, Yu Marco, & Liu, Yang. 2010. Existence of self-financing and Pareto-improving congestion pricing: Impact of value of time distribution. *Transportation Research Part A: Policy and Practice*, **44**(1), 39–51.

Ramezani, Mohsen, & Valadkhani, Amir Hosein. 2023. Dynamic ride-sourcing systems for city-scale networks - Part I: Matching design and model formulation and validation. *Transportation Research Part C: Emerging Technologies*, **152**, 104158.

Seo, Toru, & Asakura, Yasuo. 2022. Multi-objective linear optimization problem for strategic planning of shared autonomous vehicle operation and infrastructure design. *IEEE Transactions on Intelligent Transportation Systems*, **23**, 3816–3828.

Xie, Jiaohong, Liu, Yang, & Chen, Nan. 2023. Two-sided deep reinforcement learning for dynamic mobility-on-demand management with mixed autonomy. *Transportation Science*, **57**(4), 1019–1046.

Yang, Hai, Shao, Chaoyi, Wang, Hai, & Ye, Jieping. 2020. Integrated reward scheme and surge pricing in a ridesourcing market. *Transportation Research Part B: Methodological*, **134**, 126–142.