A Continuum Approximation Approach for Electric Vehicle Public Charging Infrastructure Planning

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1 INTRODUCTION

In this study, we propose a novel continuum approximation (CA) approach to optimally plan urban public charging infrastructure for electric vehicles (EVs), considering spatial heterogeneity, serviceability, and region-wide equity. We analytically estimate waiting and travel times to charge with queueing theory and evaluate serviceability over a planning region with spatial heterogeneity. The analytical model explicitly identifies three groups of factors to determine serviceability: (i) planning factors, including the spatial heterogeneity of station density and number of chargers per station; (ii) operational factors, such as station assignment rules; and (iii) exogenous factors, including charging demand, roadway network, and traffic conditions. We validate our planning framework via a numerical example and a case study focusing on New York City.

2 PROBLEM DESCRIPTION

The objective is to plan a public EV charging system that maximizes city-wide serviceability within a given budget. Here, serviceability can be improved by reducing users' time delay costs for using chargers. We consider a continuous two-dimensional service region A, a set of coordinates $(x, y) \in \mathbb{R}^2$. To address spatial heterogeneity, we define the infinitesimal unit analysis section at (x, y), with width $dx \longrightarrow 0$ and height $dy \longrightarrow 0$. In the CA framework, we assume that a value specified at (x, y) reflects an average situation nearby, and the periphery is relatively homogeneous. The two planning variables in this study are the density of the charging station at (x, y), $s(x, y) \in \mathbb{R} \geq 0$ [stations/km^2], and the number of chargers per station $h(x, y) \in \mathbb{N}$ [chargers/station]. The average charging demand per hour for a unit area at (x, y) is denoted by $\Lambda(x, y)$ [veh/hr/km^2], and the average travel speed on the road at (x, y) is $v(x, y)$ [km/hr].

Since the value of time for waiting in queue and for travel may be different, we use a weighted delay metric as an integrated measure. This metric constitutes the weighted sum of waiting time and travel time, incorporating a weight factor to consider the extra costs for electricity. We consider two station assignment scenarios when a charging demand occurs. An EV chooses (i) the station with the shortest travel time (**Assignment Scenario 1**) and (ii) the station with the shortest waiting time within a specific search area, defined by the boundary that the EV can reach within a certain time (**Assignment Scenario 2**).

2.1 Waiting Time

We utilize the M/G/k queueing model with an approximation to estimate the expected waiting time in a queue. In **Assignment Scenario 1**, each station operates as a separate queueing system with $k(x,y)$ being equal to $h(x,y)$, and the average arrival rate at a system, $\lambda(x,y)$ [veh/hr] can be approximated as $\Lambda(x,y)/s(x,y)$. In **Assignment Scenario 2**, with a given search area size, a [km^2], each search area is approximated to a queueing system with $k(x,y)$ equal to $s(x,y)\cdot h(x,y)\cdot a$, and $\lambda(x,y)$ is the average arrival rate of all stations in the search area, expressed as $\Lambda(x,y)\cdot a$. In both assignment scenarios, the waiting time is expressed as $t_{waiting}(s(x,y),h(x,y),x,y)$ hereafter to clarify that this is influenced by the planning factors at (x,y).

2.2 Travel Time

In **Assignment Scenario 1**, if stations are positioned on a square lattice at 45 degrees, the expected travel time, $t_{travel}(x,y)$ [hr/veh] is calculated as $\sqrt{2}/3\cdot(1/v(x,y)\sqrt{s(x,y)})$ (Daganzo and Ouyang, 2019). In **Assignment Scenario 2**, given that the stations are uniformly distributed within the search area, we assume that the probability of an EV being assigned to the station with the shortest queueing delay is the same for all stations. If the search area shape is a square slanted at 45 degrees, the expected travel time is calculated as $\sqrt{2}/3\cdot(\sqrt{a}/v(x,y))$. In both assignment scenarios, the travel time is influenced not by charger number per station $h(x,y)$. Thus, we will express it as $t_{travel}(s(x,y),x,y)$ without $h(x,y)$ as an input to specify the sole influential planning factor on it, $s(x,y)$.

3 OPTIMAL PLANNING

3.1 Optimization Problem Formulation

The objective is to minimize the total average weighted delay in a city per charging vehicle, $T_{delay}(\boldsymbol{s},\boldsymbol{h})$ [hr/veh] over $\boldsymbol{s}$ and $\boldsymbol{h}$, equivalent to maximizing the EV charging infrastructure's level of service, as shown in Equation 1. We determine the planning factors over the region, $\boldsymbol{s}=\{s(x,y)\ \forall x,y\in A\}$ and $\boldsymbol{h}=\{h(x,y)\ \forall x,y\in A\}$.

$$T_{delay}(\boldsymbol{s},\boldsymbol{h})=\iint_{(x,y)\in A}\frac{\Lambda(x,y)}{\iint_{(x,y)\in A}\Lambda(x,y)\,dxdy}\{t_{waiting}(s(x,y),h(x,y),x,y) + t_{access}(s(x,y),x,y)\omega(x,y)\}\,dxdy. \tag{1}$$

The total cost for the installation of charging stations and chargers, denoted by $C(\boldsymbol{s},\boldsymbol{h})$ [\$], must not exceed the allocated budget. This budget constraint is expressed as:

$$C(\boldsymbol{s},\boldsymbol{h})=\iint_{(x,y)\in A}c(s(x,y),h(x,y),x,y)\,dxdy\le B, \tag{2}$$

where $c(s(x,y),h(x,y),x,y)$ is the infrastructure costs at (x,y) [\$/km^2], including the cost per

charger and station installation, and B [$] denotes the allocated budget for constructing a public charging network over the service region.
While the objective function is designed for serviceability, it is crucial to ensure region-wide equity at the same time. To prevent significantly lower serviceability in certain areas, we set constraints to limit the maximum allowable delay denoted by t^{max} [hr/veh] for each location (x, y) as:

$$t_{delay}(s(x,y), h(x,y), x, y) \leq t^{max}, \forall(x,y) \in A. \tag{3}$$

The domain of the decision variables for each (x, y), $s(x, y)$ and $h(x, y)$, are given as:

$$s(x,y) \geq 0, \forall(x,y) \in A, \tag{4}$$

$$s(x,y) \leq s^{max}(x,y), \forall(x,y) \in A, \text{ and} \tag{5}$$

$$h(x,y) \in \{1,2,\dots,h^{max}(x,y)\}, \forall(x,y) \in A, \tag{6}$$

where $s^{max}(x, y)$ is the upper bound of $s(x, y)$ and $h^{max}(x, y) \in \mathbb{N}$ is the maximum possible number of chargers per station at (x, y). Only in **Assignment Scenario 2**, the station accessibility within the search area is secured by:

$$s(x,y) > \frac{1}{a}, \forall(x,y) \in A. \tag{7}$$

3.2 Solution Method

As the number of decision variables is infinitely large in the CA framework, it becomes necessary to decompose the continuous area into finite space elements for feasible calculation. Furthermore, due to pooled budget constraints in Constraint 2, the computation time needed to solve the optimization problem has become excessively long. To address this issue, we decompose the original problem into multiple independent location-specific problems. As the objective function is monotonically decreasing and the budget constraint is monotonically increasing with $s(x, y)$ and $h(x, y)$, the budget constraint is tight until $s(x, y)$ and $h(x, y)$ reach their upper bounds. Therefore, the dual variable multiplied by the budget constraint in the dual problem of the above can be converted into a given relative weight factor Δ ($0 \leq \Delta \leq 1$) in the objective function. This budget-free problem does not involve any pooled constraint, and $t_{waiting}(x, y)$, $t_{travel}(x, y)$, and $c(x, y)$ are independent of $s(x', y')$ and $h(x', y')$ if $(x', y') \neq (x, y)$. Therefore, the problem is decomposable into the following location-specific problem for each (x, y):

$$\begin{aligned}\min_{s(x,y),h(x,y)|\Delta} (1-\Delta)\frac{\Lambda(x,y)\eta(x,y)}{\sum_{(x,y)\in A'}\Lambda(x,y)\eta(x,y)}\{t_{waiting}(s(x,y),h(x,y),x,y) \\ + t_{travel}(s(x,y),x,y)\omega(x,y)\} + \Delta c(s(x,y),h(x,y),x,y)\eta(x,y).\end{aligned} \tag{8}$$

If the optimal set of $s(x, y)$ and $h(x, y)$ for given Δ is $s^*(x, y|\Delta)$ and $h^*(x, y|\Delta)$, the total user delay and agency budgets for that specific Δ are given as follows, respectively:

$$T^*_{delay}(\boldsymbol{s}^*, \boldsymbol{h}^*|\Delta) = \sum_{(x,y)\in A'} \frac{\Lambda(x,y)\eta(x,y)}{\sum_{(x,y)\in A'}\Lambda(x,y)\eta(x,y)} t_{delay}(s^*(x,y|\Delta), h^*(x,y|\Delta), x, y), \tag{9}$$

$$C^*(\boldsymbol{s}^*, \boldsymbol{h}^*|\Delta) = \sum_{(x,y)\in A'} c(s^*(x,y|\Delta), h^*(x,y|\Delta), x, y)\eta(x,y). \tag{10}$$

4 NUMERICAL EXAMPLE AND CASE STUDY

We define a high-density area, such as the Central Business District (CBD) at the center of the test network, with homogeneous and high demand, and outside the high-density area, the demand decreases linearly. Moreover, the high population density in the high-density area leads to reduced vehicle speeds and increased installation costs for stations. Figure 1(a) illustrates the minimum user delay values under binding agency budgets, interpretable as the Pareto frontiers between user and agency costs. In both scenarios, we observe that as agency budgets increase, user delay decreases. For further analysis, we selected one specific budget case to highlight optimal planning under a limited budget, as shown in Figure 1(b). We find that solution varies with spatial heterogeneity, high-density areas require more stations and chargers, while low-density areas need some for equity concerns. Compared to **Assignment Scenarios 1** and **2** exhibit a more uniform station density due to the pooled charger set within the search area providing higher resilience against demand fluctuations compared to the charger set at the nearest station in **Assignment Scenario 1**.

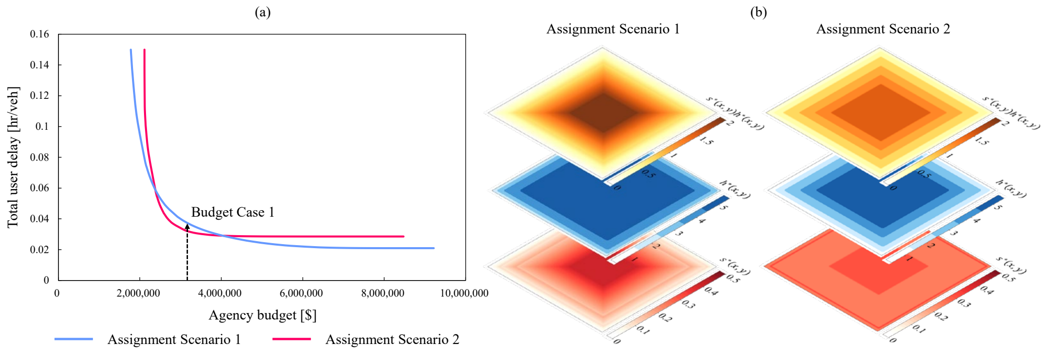

Figure 1 – *(a) Optimal total user delay under agency budget (b) Optimal station density, charger number per station, and total number of chargers per unit area*

Figure 2 depicts the results of applying the proposed framework to the five boroughs of New York City (The Bronx, Brooklyn, Manhattan, Queens, and Staten Island) (**Case 1**) and solely to Manhattan (**Case 2**) under a specific budget for each case. In **Case 1**, compared to the other boroughs, Manhattan, characterized by high density, requires higher station density and more chargers per station. When we focus on the optimal planning solution in Manhattan (**Case 2**), similar to Figure 1, it is evident that a higher density of charging points is needed in high-density areas. **Assignment Scenario 2** exhibits a higher and more uniform station density and a low number of chargers per station across the entire network compared to **Assignment Scenario 1**.

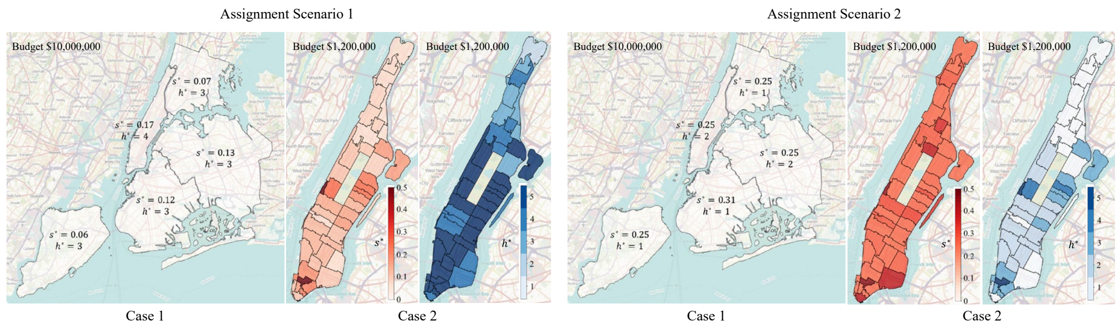

Figure 2 – *Optimal planning solutions for five boroughs of New York City (**Case 1**) and Manhattan (**Case 2**)*

5 REFERENCES

References

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