Game-Theoretical Model of Pricing in Multi-Modal Transportation Systems with Public and Private Players

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1 INTRODUCTION

Modern-day transportation systems involve many stakeholders who influence the overall impacts of each other's decisions. Therefore, it is imperative to model all pertinent stakeholders to reasonably predict scenarios and facilitate informed decision-making among them. In this abstract, we present a Game-theoretical model addressing these motivations. It can be applied to pricing decisions of public and private players/stakeholders in multi-modal transportation systems with different types of travelers.

2 METHODOLOGY

2.1 User Equilibrium Model

The user-equilibrium model lies at the core of the game-theoretical model of the overall transportation system. After all public and private players have decided their pricing/toll values for the current game iteration, users (general travelers) react by deciding: 1) whether to travel or not. If yes, then a) which mode or combination of modes to take b) which physical path to take. Supply side of the transportation system consists of multiple modes, multiple mode-classes and a transportation network, and demand side consists of multiple user-classes each having a variable travel demand, specific Value of Time (VOT) and owning a specific set of mobility resources. The concerned user decisions are, then, modelled by applying a variant of Wardrop's equilibrium principle to represent the combined demand and route-choice equilibrium of the users.

2.2 Government Model

In our model, public players like different governments may exert control over the transportation system by imposing a wide variety of tolls. Tolling instruments available in our model are:

1. Tolls for entering a particular set of physical links. A cordon toll scheme may also be designed using this type of tolls instruments.
2. Tolls charged per km travelled on a particular set of physical links e.g., infrastructure usage charge etc. For both (1) and (2), each mode, mode-class and user-class (including particular OD pairs or ownership classes) can be individually targeted.
3. Tolls charged for a particular path based on characteristics of the path other than the links it uses e.g., a trip-based bus ticket.
4. Tolls charged from or subsidies provided to private Mobility Service Providers (MSPs) based on links served or paths offered by them to users.

The objective of the government type player involves the total benefit to its constituency from travel, total travel costs of its constituency including external travel costs, total external costs to its constituency including pollution and accident costs and the total revenue collected within its jurisdiction. The government type player optimizes its objective function with its toll instruments as control variables and the user-equilibrium model as a constraint. This is basically a Stackelberg game with government type player as a leader and the users collectively being followers engaged in a Nash game amongst each other.

2.3 Fixed-line MSP Model

In this abstract, we only model private MSPs which provide either a fixed-line mobility service e.g., a bus/tram service etc., or micro-mobility services with a fixed pick-up point for the morning peak e.g., a bike sharing service from a train station. These players have similar instruments as the ones discussed before except that they are interpreted as tickets instead of tolls. However, their objective function is purely a profit. They solve a similar optimization problem as the government type player being subject to the user-equilibrium model.

2.4 Optimistic vs Pessimistic Player

The paths flows at user-equilibrium can be non-unique in general (Boyles et al., 2022). In the context of our game-theoretical model, this means that for the same value of tolls/tickets by the government and private players, there may exist multiple equally rewarding options to respond for users. If these multiple options of users lead to different objective function values for the players deciding their prices, it becomes problematic for them to assess the optimality of a given toll/ticket value. This issue is discussed more in detail in one of our forthcoming papers (Malik and Tampère, 2024). Thus, to specify the problem further, we introduce an attribute of being either optimistic or pessimistic for each player. An optimistic player will assume that the users will break ties between their multiple options in a way that is most desirable for the player whereas a pessimistic player will assume that the users to do so in a way that's least desirable for itself. From a mathematical perspective, the problem of a pessimistic player is much harder to solve than the optimistic player.

2.5 Multi-Player Games

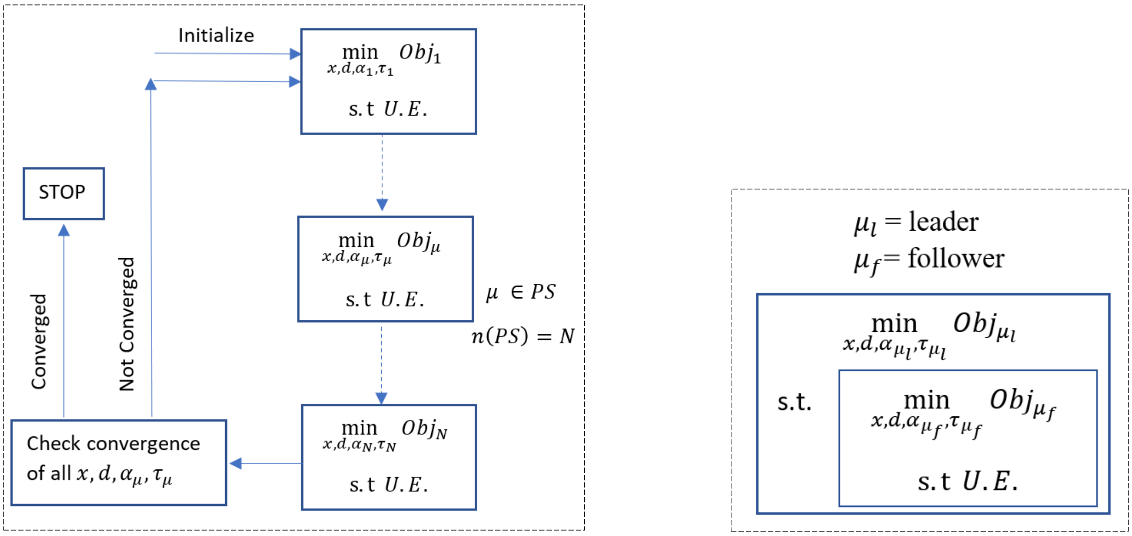

Figure 1*: Representation of solution process: NC game (left) and Stackelberg game (right)*

In this abstract, we discuss two types of games between a government and an MSP: Nash-Cournot (NC) Game and Stackelberg Game. In NC game, the government and the MSP are at the same level. They both take the other's tickets/tolls as given and solve their optimization problems with the user-equilibrium model as constraints. Each of them individually still acts as a Stackelberg leader over the users. In the Stackelberg Game, one of the players takes a leadership position. The leader optimizes its objective function constrained to the follower optimizing its objective function constrained to the user-equilibrium. This game becomes a tri-level game in which the leader decides its prices anticipating the prices of the follower, which in turn, decides its price anticipating the travel choices of the users. These games are solved using the schemes of Figure 1 where x and d refer to path and demand flows of users respectively, α_{μ} and τ_{μ} refer to the per km and entry-based price instruments of player μ respectively.

3 RESULTS

3.1 Case Study 1

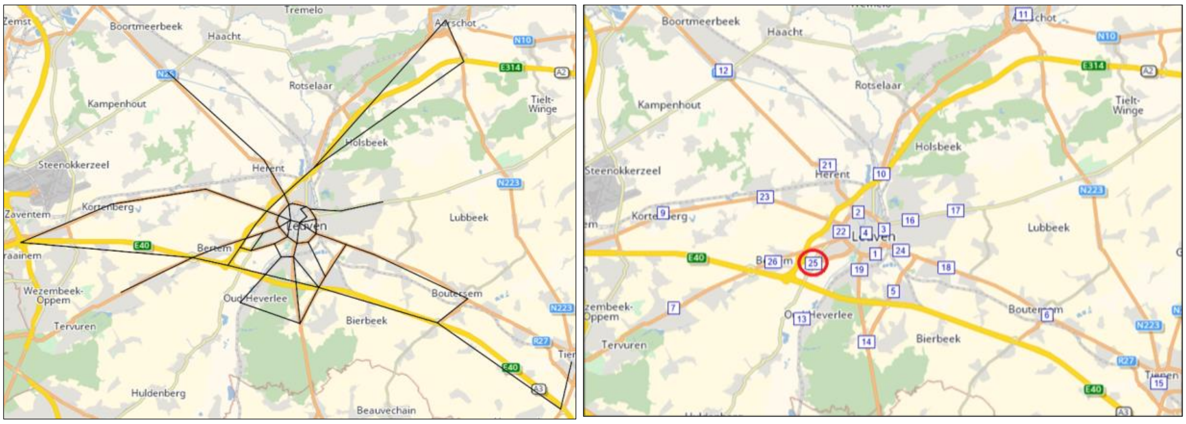

Figure 2: *Network Physical Links (left) and centroids (right). Mobility hub is shown in red circle.*

We applied our model to a pseudo real case-study based on the city of Leuven. The network with physical links and centroids is shown in Figure 2. We considered the modes of personal car and an MSP offered bus service. For the mode of personal car, we considered the mode-classes of conventional and electric cars. Moreover, we considered a mobility hub in the south-west of Leuven where a switch from personal cars to buses may take place; thereby providing mixed-mode options to the users. The link and demand side parameters were estimated using available data from the Flemish Departement Mobiliteit en Openbare Werken (MOW), literature and google maps for a peak hour travel. Further, four user-classes were modelled based on whether users are old or young and whether they own an electric or a conventional car, and a total of 1072 paths were modelled. Two pricing instruments were considered: 1) A cordon toll on conventional cars (t_{CVcar},) to enter the inner city of Leuven 2) A trip-based bus ticket (t_{bus}). We solved for four distinct game settings:

1. Central case: A central player decides both t_{CVcar} and t_{bus}, and optimizes the joint objective function.
2. NC game: The Leuven government, with its jurisdiction and constituency comprising of the Greater Leuven Area, controls t_{CVcar} and the bus provider controls t_{bus}.
3. Stackelberg game 1: The Leuven government acts as a leader.
4. Stackelberg game 2: The bus provider acts as a leader.

Table 1: *Results of Case Study 1*

Metric/Scenario	**Central case**	**Central case: Worst**	**NC game**	**Government as leader**	**MSP as leader**
Computation Time (s)	381	476	3093	5561	10527
Obj_c (Euros)	-435222	-434565	-434231	-434877	-434231
Obj_G (Euros)	-434985	-434985	-433480	-434169	-433480
Obj_{MSP} (Euros)	1833	1833	1319	1361	1319
t_{CVcar} (Euros)	1.05	1.05	0.98	0.44	0.98
t_{bus} (Euros)	0	0	1.68	0.81	1.68

Both players were assumed to be optimistic for this case-study. The results are shown in Table 1. A negative objective function denotes a positive profit.

3.2 Case Study 2

In this case study, we re-solved the central case and the Nash-Cournot game with the assumption that the central player and the Leuven government are pessimistic respectively in these cases. The results are shown in Table 2.

Table 2:*Results of Case Study 2*

Metric/Scenario	Central case	Central case: Best outcome	NC game
Computation Time (s)	10816	10908	25264
Obj_c (Euros)	-434781	-435062	-433809
Obj_G (Euros)	-434424	-434706	-432984
Obj_{MSP} (Euros)	1714	1714	1245
t_{CVcar} (Euros)	1.27	1.27	1.22
t_{bus} (Euros)	0	0	1.86

4 DISCUSSION

Some interesting observations from the results are:

1. When the central player controls both t_{CVcar} and t_{bus} (in both optimistic and pessimistic cases), it is optimal to provide a free bus service. The revenue made from bus tickets does not compensate for the increase in total costs due to more cars.
2. The central objective function is the best in the central case and any form of competition can only worsen it. As soon as the bus provider acts independently, it will always ask a bus ticket to lower its losses.
3. In Case Study 1, when government acts as a leader, it lowers t_{CVcar} as compared to the central and the NC game as it anticipates that any increase in t_{CVcar} will be accompanied by an increase in t_{bus} by the bus provider which discourages the users from traveling and hence worsens its objective function. By choosing a lower t_{CVcar}, it achieves an extra 689 Euros for itself as compared to NC game; however, due to the positive t_{bus}, it still can't match the central case.
4. In Case Study 1, when the bus provider (MSP) acts as a leader, it can't do better than the NC game because the government as a follower is not highly influenced by its bus ticket.
5. In Case Study2, when the central player is pessimistic, it sets a higher t_{CVcar} as compared to when it is optimistic in Case Study 1. In optimistic case, it believes that if presented with two equal cost paths, conventional cars will take the path that causes lower pollution and accident costs, and this allows it to be a bit more lenient with the toll value. In the pessimistic case, it sets a higher toll to be sure. In the optimistic case, the central player exposes itself to the possibility of achieving a payoff of 435,222 Euros, albeit with the risk of only attaining 434,565 Euros (Central case: Worst outcome). Conversely, in the pessimistic case, it ensures a minimum payoff of 434,781 Euros while still retaining the opportunity to achieve 435,062 Euros (Central case: Best outcome).

References

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