On numerical investigation of epidemics transport dynamics using a coupled PDE crowd flow - epidemics spreading dynamics model

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1 INTRODUCTION

Macroscopic modeling of the dynamics of epidemics transport is of significant importance as such dynamic models can be used for prediction of epidemics spreading in different areas, while keeping a relatively lower computational burden as compared with detailed models accounting for individual-to-individual interactions. For this reason, there are coupled models that describe epidemics spreading over different geographical regions, such as, for example, [1], [2], [3]. Although such models are useful for describing the epidemics transport effect at higher levels, e.g., at a city or prefecture levels, they may not describe epidemics transport at the level of a specific closed space, e.g., a metro corridor or a conference venue. For this reason in [4] and [5] coupled crowd flow - epidemics spreading dynamics models are presented, which describe epidemics transport via accounting for the effect of people movements in certain spaces.

In the present abstract we employ the model we present in [6], which employs a different crowd flow model component than [5], as well as it accounts explicitly for ventilation rate dynamics. In particular, we present a model consisting of three parts, a crowd flow dynamics component, an epidemics spreading model, and an equation that provides the velocity field due to ventilation. Using the numerical scheme we present in [6] we perform different tests than in [6]. Specifically, we present here preliminary and simpler-to-implement tests, in comparison with [6], in which the crowd is moving towards an exit that is assumed to be the whole right boundary of the computational domain (instead of, e.g., only a specific part of it, which would be more practically realistic). Detailed explanations and results are included in [6].

2 COUPLED CROWD FLOW - EPIDEMICS SPREADING MODEL

2.1 Crowd Flow Model

By denoting as $\mathbf{x} = (x, y) \in \Omega$ the spatial variables and $t > 0$ the time, we define as $\rho(x, y, t)$ the pedestrian density, while as $u(\mathbf{x}, t)$ and $v(\mathbf{x}, t)$ we denote the $x-$component and $y-$component of the velocity vector $\mathbf{v}$, respectively. The model equations can be written as

$$\rho_t + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}$$

$$(\rho \mathbf{v})_t + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v} + P(\rho)) = \frac{1}{\tau} \rho \left(V(\rho) \vec{\mu} - \mathbf{v} \right), \tag{2}$$

where $P(\rho) = \rho C_0^2$ is an internal pressure function, with C_0^2 being constant representing an anticipation factor, μ is the desired direction vector, and τ is relaxation time. For the speed-density relation we implement the following relation

$$V(\rho) = u_{\max} e^{-\alpha(\rho/\rho_{\max})^2}, \tag{3}$$

where $\alpha > 0$ is a constant, $u_{\max}$ is the free-flow velocity, and $V(\rho_{\max}) \approx 0$ with $\rho_{\max}$ being the congestion density at which the motion is hardly possible. With regard to the desired direction of motion $\vec{\mu}$, we obtain $\vec{\mu}$ by solving an eikonal equation as in, e.g., [7].

2.2 Epidemics Spreading Model

The model utilized here is from [5] and is based on a macroscopic version of a SEIS model where each type of pedestrian (SEI) moves with the crowd speed $\mathbf{v}$. The density of each type of pedestrian satisfies then the following

$$\rho_t^S + \nabla \cdot (\rho^S \mathbf{v}) = \kappa \rho^I - \beta_I \rho^S, \tag{4}$$

$$\rho_t^E + \nabla \cdot (\rho^E \mathbf{v}) = \beta_I \rho^S - \theta \rho^E, \tag{5}$$

$$\rho_t^I + \nabla \cdot (\rho^I \mathbf{v}) = \theta \rho^E - \kappa \rho^I, \tag{6}$$

where ρ^S, ρ^E, and ρ^I are densities of the susceptible, exposed, and infected pedestrians, respectively, satisfying $\rho = \rho^S + \rho^E + \rho^I$. Further, β_I is the infection rate, κ is the recovery rate and θ is the rate with which exposed persons are becoming infected. However, on the time scales under consideration, κ and θ are very small and set to zero in our numerical simulations. We compute $\beta_I = i_0 \beta(\mathbf{x}, t)$, where $\beta(\mathbf{x}, t)$ is the solution of the following drift-reaction-diffusion PDE in 2D following [5] as

$$\beta_t + \nabla \cdot (\beta \mathbf{U}_G) = \nabla \cdot (\sigma \nabla \beta) - \nu \beta + \frac{\rho^I}{\rho}, \tag{7}$$

and $\mathbf{U}_G$ is a given velocity field of the surrounding air-flow in the computational region. Variable σ is effective turbulent viscosity for the aerosol and the term $-\nu\beta$ models the fact that aerosol particles are settling due to the gravitational force, while the parameter i_0 is determined by the infectivity [5]. To produce a steady velocity field $\mathbf{U}_G$ we employ the solution of a Laplace equation in 2D.

2.3 Numerical Implementation of the Model

The details of the numerical implementation scheme are omitted due to space limitation and since they are repeated in [6]. We briefly discuss here this implementation. The crowd flow model is numerically solved employing a finite-volume scheme, using the Roe Riemann solver, see, for example, [8]. For the direction vector we solve the respective (eikonal) Hamilton-Jacobi-type equation using the Fast-Sweeping Method, see, e.g., [9]. The epidemics transport equations are solved using a finite-volume scheme with Rusanov flux, see, e.g., [8]; while the equations for aerosol dynamics and ventilation rate are solved using finite-difference schemes, see, e.g., [8].

3 SIMULATION RESULTS

We consider a walking facility of size $[0, 50m] \times [0, 20m]$ with an exit being, for simplicity as a preliminary result, the whole right boundary. The initial density is $\rho_0 = 1ped/m^2$ in the region $[0, 20m] \times [0, 20m]$ with $v_0 = 0$. We perform simulations with the velocity field $\mathbf{U}_G$ shown in Figure 1 with inflow velocity $u_{in} = 10m/s$.

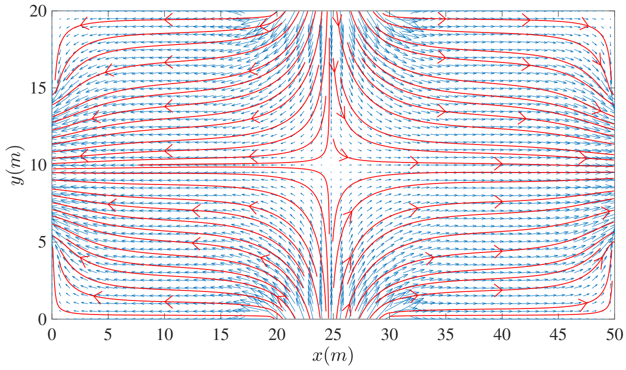

Figure 1 – *Ventilation velocity field considered.*

Figure 2 illustrates epidemics transport at two different time instances $t = 10s$ and $t = 20s$. We observe that the number of exposed individuals increases as the crowd is moving towards the exit, also depending on the dynamics of the infection rate β.

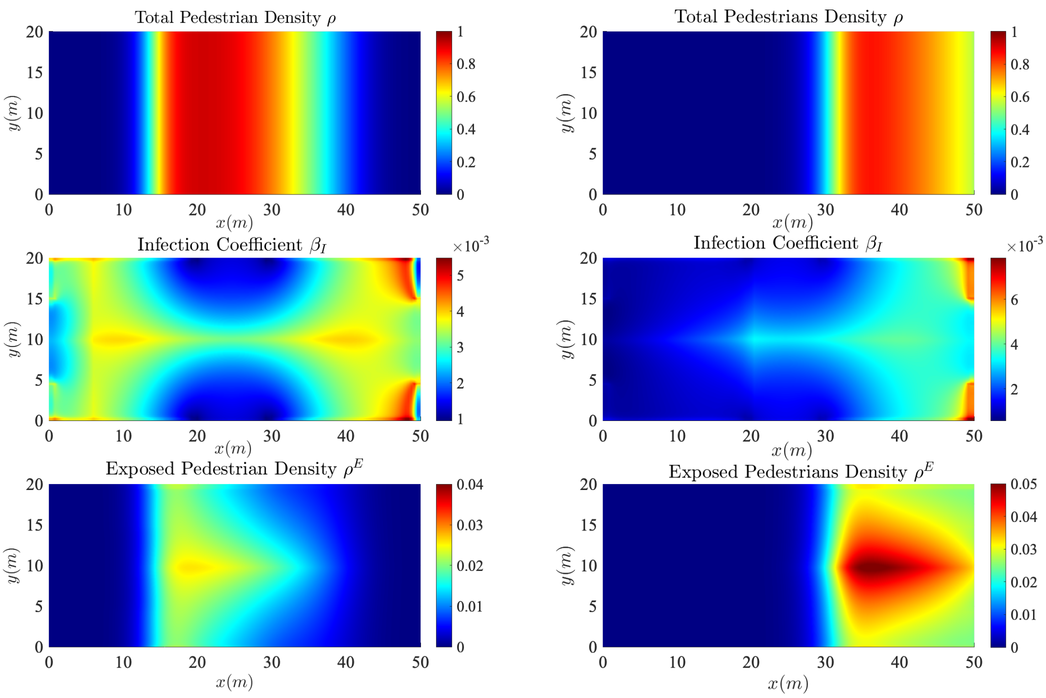

Figure 2 – *Total density, infection rate coefficient, and density of exposed individuals at two different time instants, namely, $t = 10s$ (left) and $t = 20s$ (right).*

4 CONCLUSIONS AND CURRENT WORK

We presented numerical tests implementing an epidemics transport model that involves a crowd flow dynamics and an epidemics spreading dynamics component. We studied the behavior of epidemics transport under movements of pedestrians in a closed area, in the presence of ventilation. We are currently further validating the model in various numerical scenarios, as well as we investigate potential design of real-time control strategies for the ventilation rate to reduce spreading, which may account for the dynamics of speed and density in time and space.

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