Nonlinear string stability analysis of car-following models: metastability thresholds and rear-end collisions

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1 INTRODUCTION

An experiment conducted in the ZalaZONE proving ground by means of ten commercial high-end vehicles showed a clear pattern of amplification of leader's disturbances potentially leading to rear-end collisions, under several kinematic conditions and system settings (Ciuffo el al, 2021). The study concluded that with more vehicles equipped with ACC, serious safety problems will arise dramatically. As a result of the emerging discussion, an amendment to UN Regulation on automated lane keeping systems (ALKS) introduced string stability as a requirement for these systems (UN R 157 Amendment 4, 2023). The study of string stability of ACC controllers and car-following models, that emulate longitudinal dynamics of automated vehicles (AV) and human-driven vehicles (HDV), respectively, is crucial. String stability is usually investigated in a *linear setting* (Wilson, 2001; Wilson, 2008; Ward, 2009; Wilson and Ward, 2011; Treiber and Kesting, 2011; Treiber and Kesting, 2013; Ngoduy, 2013; Ngoduy, 2015; Ngoduy et al., 2019; Monteil et al., 2019; Montanino et al., 2021; Montanino and Punzo, 2021; Bouadi et al., 2022; Wang et al., 2022). However, the results are only valid for 'small perturbations' (there is no definition of a 'small perturbation'), so it is dangerous to consider only the linear stability analysis. In fact, it can happen that a non-linear model/controller, stable according to a linear analysis, i.e. subject to 'small' disturbance intensities, becomes unstable for larger intensities. When this happens, model/controller is *metastable*. Therefore, it is crucial to investigate the string stability in a nonlinear field. Very few studies investigated this topic, we found two distinct approaches. The first approach uses an analytical method to study the nonlinear string stability, via the modified Korteweg-de-Vries (mKdV) equation (Nagatani, 1998; Ge et al., 2005; Zhu and Dai, 2008; Jin et al., 2011; Gong and Zhu, 2022), or a more general approach based on the $\mathcal{L}_\infty$ characterization of string stability (Monteil et al. 2019). The second approach is characterized by simulation, to the best of our knowledge, the only work was Treiber and Kesting (2011), that characterized the metastability region of the IDM parameter space as a function of the equilibrium spacing. To assess the consistency between analytical results of a linear string stability analysis and those by numerical simulations of a model subjected to a perturbation at equilibrium (a deceleration between 0.1 and 3 m/s^2), also in Sun et al. (2018) a metastable behaviour of the IDM was characterised, though not explicitly defined as such. In all these studies, however, the validity limit of the linear string stability results, i.e. a metastability threshold, was not identified. Therefore, a first contribution of this paper is to quantify a *metastability threshold* as a function of the disturbance type and intensity, the traffic conditions and the car-following model applied. A second contribution of the work concerns the safety implications of a string unstable behaviour, with the aim to verify if a string unstable car-following model can produce crashes in simulation, and thus can be adopted for traffic safety analyses. Eventually, as delays are acknowledged to contribute to linear string instability, and particularly perception delay is a key factor also for rear-end collisions (see e.g., Treiber et al., 2007; He et al., 2022), we studied how the metastability behaviour of car-following models changes when a model is augmented by a *perception delay*. The study was carried out on two well-known nonlinear car-following models, the IDM (Treiber et al., 2000) and Gipps' model (Gipps, 1981) and two more variants obtained by constraining the output decelerations not to exceed limits set in ISO15622 (2018) and augmenting the models by a perception delay.

2 METHODOLOGY

Figure 1 describes the methodological flow applied in this work to study the string stability of car-following models in a nonlinear setting, under the assumption of homogenous flow. First a linear string stability analysis is performed via analytical computation of the $\mathcal{L}_\infty$ system norm. In fact, metastability is a condition which arises for linearly stable parameter sets only. Sobol sequences of quasi-random numbers (Sobol, 1967) are used to sample model parameter values from independent uniform distribution, and to characterize the stability behaviour in the full parameter space. Then, linearly stable platoons are simulated subjected to disturbances of increasing intensity. The analysis is carried out at different equilibrium speeds (10;15;20 m/s), for different input disturbance types (see Figure 2), and for increasing perception delays.

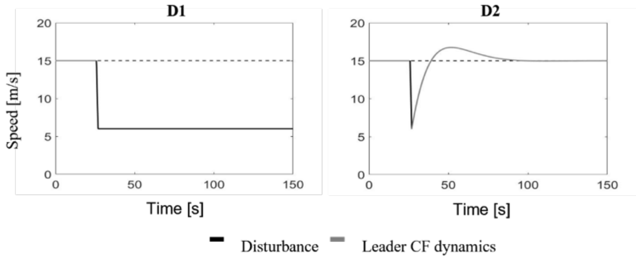

Figure 1- *Leader's speed profiles subjected to two types of deceleration disturbance (D1 and D2).*

If the following condition is verified, the platoon is <u>*metastable*</u>:

$$\mathcal{L}_\infty \leq 1 \ and \ \left(\frac{\|\dot{y}_n\|_\infty}{\|\dot{y}_3\|_\infty}\right)_{sim} > 1$$

where $\mathcal{L}_\infty$ is the ∞-norm of the linearized car-following model, $\|\dot{y}_n\|_\infty$ and $\|\dot{y}_3\|_\infty$ are the maximum deviations from the equilibrium of the speeds of, respectively, the last and third vehicle in a platoon. An analogous condition can be formulated on spacing.

3 RESULTS

3.1 Model without perception delay.

For both IDM model variants, the metastability is more sensitive to disturbance D1 than D2. In particular, the metastability threshold for disturbance D1 is always lower or equal to -0.6 m/s^2 for both model variants, whatever the value of the equilibrium speed. The total number of metastable platoons for disturbance D1 is in the range 14-17% of the number of $\mathcal{L}_\infty$ linearly stable platoons, for both model variants and tested equilibrium speeds. The deceleration constraint was found to have marginal effect on the metastability of the IDM, in fact when D1 is applied the percentage is almost the same. If D2 is applied, a slight increase in metastability is observed. Concerning Gipps' model there is no sensible difference between the two model variants. The metastability threshold is significantly higher than that of the IDM (7 vs. 0.6 m/s^2). It is worth nothing that for the highest equilibrium speed there are no metastable platoons, while for the lowest one the number of metastable platoons reaches the 67% of all platoons. It is worth noting that a major criticism to Gipps' model is that it becomes linearly unstable at high-speed values, contrary to what is observed in real traffic (Wilson, 2001). Instead, the metastability of Gipps' model increases as speed decreases, consistently with observations. *This result rehabilitates Gipps' car-following model concerning the realism of its string stability behaviour*. If we adopt the IDM as a controller, it is useful to identify the frontier between stable and metastable parameter regions, which can be conveniently drawn in the a_{max}-T plane, in the 'worst case' stability behaviour (see the black line in Figure 8, left side).

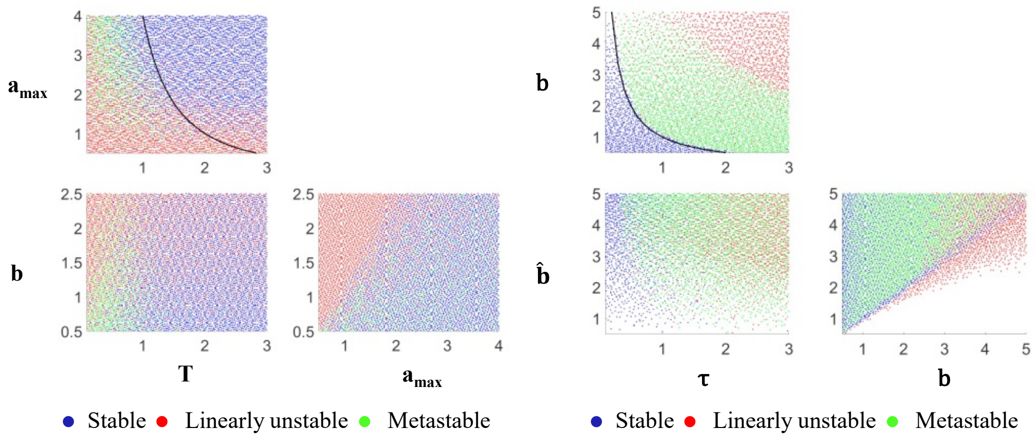

Figure 2- *Worst-case stability diagrams, for $v_{eq} \geq 10\ m/s$ (IDM left; Gipps right). Blue: stable; green: metastable; red for linearly unstable at least at one equilibrium speed.*

The region of metastable parameters is defined by the condition:

$$\begin{cases} \mathcal{L}_\infty(v_{eq}) = 1 & \forall v_{eq} \geq 10\ m/s \\ T \leq 2/\sqrt{a_{max}} \end{cases} \tag{1}$$

Similarly to the IDM case, for Gipps' model the b-τ plane is drawn in Figure 8 (right side). The region of metastable parameters for $v_{eq} \geq 10\ m/s$ is defined by the simple condition:

$$1/\tau < b < \hat{b} \tag{2}$$

As results: The 98.8% of metastable parameter sets fall in the region defined by Eq. 20, the 99.8% of metastable parameter sets falls in the region defined by Eq. 21. *Collisions did not occur neither for $\mathcal{L}_\infty$ linearly unstable platoons, nor for metastable ones* (see the blue bars in Figure 11, concerning the IDM). When a deceleration constraint was added to the models, however, collisions occurred; though the first vehicle to collide was always the one immediately behind the platoon leader. In other words, no collision occurred due to the string instability.

3.2 Model with perception delay.

By introducing a perception delay in the IDM, the percentage of metastable platoons considerably increased relative to the base model (see the left diagram in Figure 11). For a perception delay equal or higher than 1s, all $\mathcal{L}_\infty$ linearly stable platoons resulted metastable in simulation, regardless of the equilibrium speed, and almost all produced a collision. The IDM with a deceleration constraint and two Gipps' model variants (with and without a deceleration constraint) produced similar results.

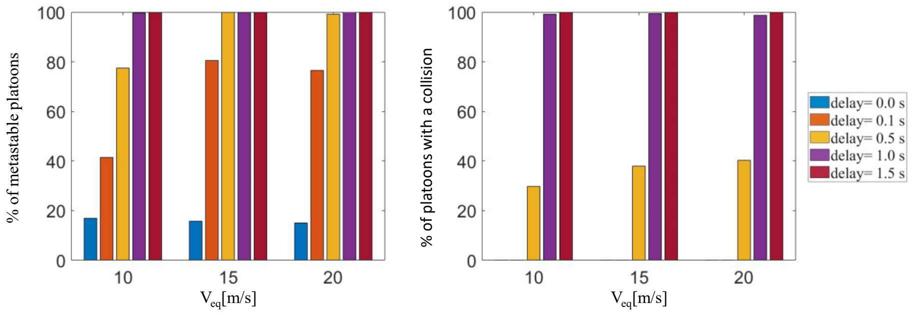

Figure 3- *Percentage of metastable platoons (left), and crashes (right), for the IDM model for different values of equilibrium speed and perception delay.*

Collisions observed in $\mathcal{L}_\infty$ linearly stable platoons simulated by the IDM augmented by a perception delay, however, were not all due to metastability, i.e., to an amplification of a disturbance, but also between the first two vehicles (for the lowest equilibrium speed).

4 DISCUSSION

In this paper, we aim to take a step forward in the study of the string stability of car-following models by questioning the assumption of "small" disturbances in linear analysis. Such an assumption reaches its limits in real traffic, where "large" disturbances, e.g. due to sharp decelerations, are common. A second problem we tackle is that analytical studies of string stability do not allow disentangling the occurrence of a rear-end collision from the amplification of a disturbance i.e., from the string instability. In fact, we provide evidence that string instability is necessary but not sufficient to cause a rear-end collision, which makes these methods unfit to traffic safety studies. A simulation approach was thus used in this work to overcome the assumption of "small" disturbances, and to investigate the relationship between string instability and collisions. It was possible to identify a so-called *metastability threshold*, i.e., the largest intensity of a perturbation for which linear stability results hold, above which a linearly string stable model exhibits an unstable behavior, i.e., it becomes *metastable*. By characterising the region of model parameters that yields a metastable behavior, analytical conditions for metastability were derived, as a function of parameters values. Among the relevant results of the analyses was the rehabilitation of the realism of Gipps' model concerning its stability properties. Indeed, it was verified that if stability is measured by means of the $\mathcal{L}_\infty$ norm, the model becomes string unstable for decreasing values of the equilibrium speed – in agreement with real traffic observations – both in the metastability region and in the linear instability region. The analysis allowed characterizing the frequency of collisions. With some surprise, no collisions due to the amplification of a disturbance were recorded both for metastable and linearly unstable parameters, even when the deceleration output of the two models was constrained not to exceed normally achievable values. Analysis results allowed us also to quantify the significant impact that the introduction of a perception delay had on metastability and on the occurrence of collisions. It is hoped that this study will break new ground in the application of car-following models to the study of road safety. The proposed methodology can be applied both to define the stability properties of an automated controller in real traffic conditions, and its impact on road safety.

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