Optimising ghost kitchen location for on-demand delivery by a Markov model for circulating couriers

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1 INTRODUCTION

On-demand meal delivery has become a prevalent feature of last-mile delivery in urban landscapes, spurred by digital platforms and temporary restaurant closures during the pandemic (Liu and Li, 2023). As the demand for online food delivery experiences continuous growth, the ghost kitchen model has emerged as a pivotal innovation for the on-demand delivery logistics domain (Statista, 2024). These kitchens, designed to optimise food delivery by eliminating front-end real estate and focusing entirely on the preparation and outbound logistics of cooked meals, significantly reduce operational costs and are integral in addressing challenges linked with road networks, congestion, risky behaviour, and excessive GHG emissions when planning poor (Lord et al., 2023, Gregory, 2021, Christie and Ward, 2019). This study extends the utility of a Markov model, developed to represent courier pickup and delivery (Bell et al, 2024), to optimise ghost kitchen location.

2 Methodology

2.1 Markov Models

Core of a Markov model is a transition matrix describing probabilistically the transition of the system from state to state, or in our case, the transition of couriers from one location (kitchen or customer) to the next. The transition probability is a function of the travel time from the current location to the next plus the dwell time at the next location and a parameter representing the intrinsic attraction of the next location. The functional form of the transition probability emerges from the entropy maximising derivation of the Markov model. The Markov model has been calibrated to the Grubhub dataset, which is publically available (https://github.com/Grubhub/mdrplib/tree/master/public_instances/0o100t100s1p100).

2.2 Problem Definition

We model kitchens and customers as nodes within a fully connected network, with kitchens (S) and customers (C) forming the complete node set ($V = S \cup C$). Node coordinates allow us to calculate straight-line distances and, assuming constant courier speeds, to derive travel times between nodes. Due to Grubhub data limitations, we do not use actual road or cycle path travel times but do include fixed dwell times at each node to represent time spent at either pickup or delivery points, integrating these into the node-to-node travel time matrix (c_{ij}). Couriers circulate based on a Markov process, with node-to-node transition probabilities determined by c_{ij}, meal delivery urgency (β), and destination demand $(q_j$, with $j \in V)$. Sequential destinations are restricted to customers after kitchens and potentially other customers or a return to a kitchen after deliveries. Paths, represented as node sequences, include all possible routes originating from any kitchen (P_i) or customer (P_j), and between any node pair (P_{ij}).

Despite theoretically allowing infinite paths, cycles like repetitive customer visits are discouraged by the model's structure and extended travel times. The model parameters, q_j and β, are calibrated by aligning the Markov model's pickup and delivery frequencies plus average delivery times with observed data. This calibrated model then enables the examination of policies, such as kitchen relocations, on demand and overall mean delivery time. Bell et al (2024) introduced two methods for calibrating model parameters. Interested readers are referred to that paper for more details. For the purposes of this analysis, we will utilise the parameters calibrated for the Grubhub dataset.

2.3 Markov Chain

Couriers are modelled as perpetually circulating. While in reality couriers may enter and exit the workforce, these changes are treated as simple replacements that do not alter the overall count of couriers. We define for $i, j \in V$:

$$a_{ij} = \begin{cases} 0 \text{ if } i = j \text{ or both } i \in S \text{ and } j \in S \\ q_j \exp(-\beta c_{ij}) \text{ otherwise} \end{cases} \quad (1)$$

The transition probability (the probability that a courier moves to $j \in V$ from $i \in V$) is:

$$t_{ij} = \frac{a_{ij}}{\sum_{k \in V} a_{ik}} \quad (2)$$

This Markov chain is *irreducible* because all nodes "communicate" with each other. Transition between kitchens can occur indirectly via customer visits, ensuring all states "communicate" (Ching and Ng, 2006). Given that this Markov chain consists of a finite number of states, it is also *positive recurrent*. An irreducible and positive recurrent Markov chain has a unique equilibrium state vector. The stability to this equilibrium for the Grubhub dataset is demonstrated in Bell et al (2024).

Let path $p' = i \to k \to l \to \cdots \to j$ and define $\pi_{p'} = q_k q_l \dots q_j$. Hence the probability that a courier chooses path $p' \in P_{ij}$ is:

$$\begin{aligned} \text{Prob}(p = p' | p \in P_{ij}) &= \frac{q_k \exp(-\beta c_{ik}) q_l \exp(-\beta c_{kl}) \dots q_j \exp(-\beta c_{mj})}{\sum_{p \in P_{ij}} \pi_p \exp(-\beta c_p)} \\ &= \frac{\pi_{p'} \exp(-\beta c_{p'})}{\sum_{p \in P_{ij}} \pi_p \exp(-\beta c_p)} \end{aligned} \quad (3)$$

For paths connecting node $i \in S$ to node $j \in C$ define:

$$LS_{ij} = \ln\left(\sum_{p \in P_{ij}} \pi_p \exp(-\beta c_p)\right), i \in S, j \in C \quad (4)$$

The mean delivery time from a kitchen at location $i \in S$ to a customer at location $j \in C$ is:

$$\mu_{ij} = \frac{\sum_{p \epsilon P_{ij}} c_p \pi_p \exp(-\beta c_p)}{\sum_{p \epsilon P_{ij}} \pi_p \exp(-\beta c_p)} = -\frac{\partial LS_{ij}}{\partial \beta} \tag{5}$$

By employing finite differencing techniques, we can approximate $\frac{\partial LS_{ij}}{\partial \beta}$ with high accuracy. After calibrating the model through the methodology outlined in Bell et al (2024), we showcase its capability to evaluate the impact on travel time of relocating a kitchen. By taking the existing Markov model parameters and recalculating the travel time matrix for the kitchen's new location, we recalculate the mean delivery time to all customers.

According to (1), as the distance between kitchen i and customer k decreases, the transition probability between these locations increases, suggesting that the optimal location of kitchen i is at a location where it is closest to all customers to minimise mean delivery time. This principle suggests that relocating kitchen i to the geometric median would be a reasonable approximation for the location with the minimum delivery time yielded by the Markov model. To identify the geometric median, we employ the Weiszfeld algorithm (Weiszfeld, 2009), known for its simplicity and effectiveness. Starting with an initial guess, the algorithm updates the median's position by computing a weighted average of the points, where the weights are inversely proportional to their distances from the median. If the median coincides with any given point, it is directly set to avoid division by zero. The process repeats until the changes are smaller than a set tolerance, indicating convergence. Algorithm 1 demonstrates the steps of this algorithm in detail.

Algorithm 1 Weiszfeld Algorithm

Require: V; k: the kitchen to move; δ: Convergence tolerance threshold; x_i: coordinates of the points.
Ensure: The estimated median point

Initialize $\bar{x}_k^* = (0, 0)$ ▷ Arbitrary initial estimate
repeat
    $\bar{x}_k = \bar{x}_k^*$
    for $i \in V/\{k\}$ **do**
        $d_{ik} \leftarrow$ The Euclidean distance between node i and $\bar{x}_k$
        if $d_{ik} \neq 0$ **then**
            Calculate weight $w_i = \frac{1}{d_{ik}}$
        end if
    end for
    Calculate $\bar{x}_k^* = \frac{\sum_{i \in V} x_i w_i}{\sum_{i \in V} w_i}$
until Euclidean distance between $\bar{x}_k^*$ and $\bar{x}_k$ becomes less than δ
Return $\bar{x}_k^*$ as the estimated median point.

3 Results and Discussion

To illustrate the accuracy of the Weiszfeld algorithm for approximating the optimal location indicated by the calibrated Markov model, we relocate kitchen 84 from its initial position at coordinates (4558, 2716). Figure 1 shows the mean delivery time surface estimated by the Markov model using the q values estimated for the Grubhub dataset when $\beta = 3.8$. A grid search suggests that the optimum location for the Markov model lies near coordinates (8000, 7000). Algorithm 1 found the geometric median to be at coordinates (8307.4, 7059.1), which is a good approximation. The full paper will further explore the use of the Weiszfeld Algorithm, and modifications to it like replacing d_{ik} by μ_{ik}, for optimally locating ghost kitchens. In this way, we will tailor the Weiszfeld Algorithm to more closely correspond to the Markov model.

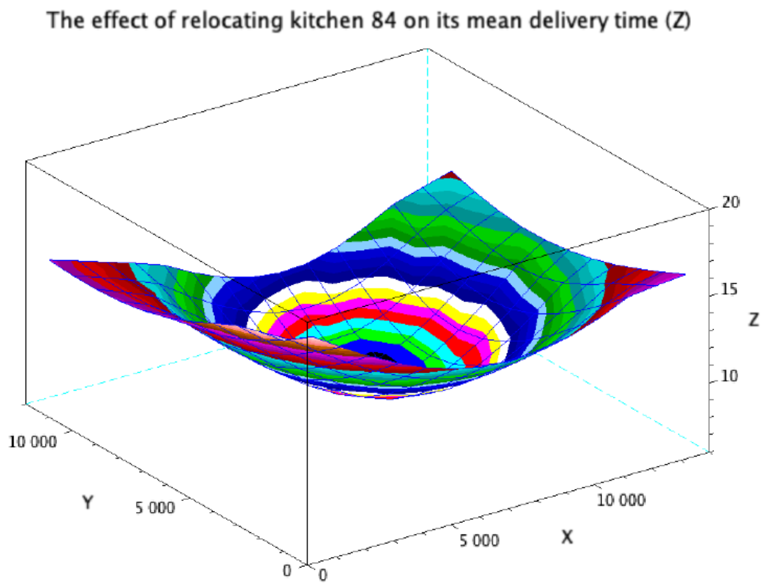

Figure 1 – *The mean travel time surface of kitchen 84, employing an exhaustive search across the X and Y plane.*

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