Optimal demand-responsive connector design: Comparing a fully-flexible routing strategy and a semi-flexible routing strategy

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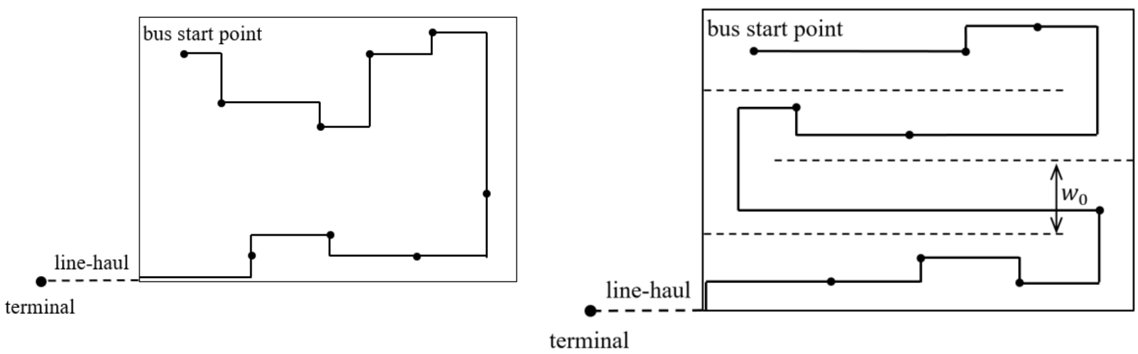
1 INTRODUCTION

Demand-Responsive Connector (DRC) provides personalized door-to-door transport, enhancing passenger convenience and operational efficiency. It employs two primary strategies: fully-flexible and semi-flexible routing. Fully-flexible routing uses the optimal TSP path, requiring all passenger requests to be known before dispatching the bus (see Fig. 1a). Alternatively, semi-flexible routing is a heuristic approach where the bus traverses the zone longitudinally along the swath and deviate laterally to pick up the demand (see Fig. 1b).

Previous studies on DRC design are flawed as they have not properly distinguished these two strategies. In this paper, we correctly modelled these two strategies and compared them under various scenarios. To improve the validity of model outcomes, we also give a more precise calculation of the local tour length. Lastly, we refine and advance the modelling of stochastic demand in DRC design optimization, markedly enhancing modelling accuracy.



(a) Optimal TSP tour under fully-flexible routing

(b) Semi-flexible tour (Daganzo, 1984)

Figure 1 – *Two routing strategies*

2 METHODOLOGY

2.1 Experimental set-up

Fig. 2 shows the layout for DRC service. We only look at a quarter of the surrounding rectangular area for our designs. The area is divided into equal rectangles for either routing strategy, and each is labeled by its position. We assume that the demands of patrons in both directions follow spatial Poisson process with a uniform rate. The decision variables including the number of zones, headways, the bus capacity and length of swaths.

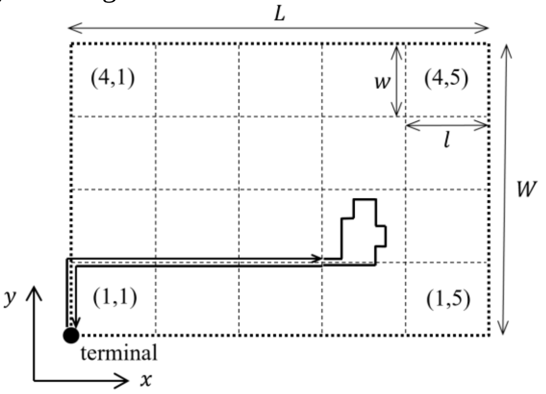


Figure 2 – *DRC service network*

2.2 Fully-flexible routing strategy model

We used a Monte Carlo simulation to predict the optimal route lengths with few passengers. The results are shown as follows:

$$k^* = \begin{cases} (-0.0807 - 0.0061S)q^2 + (0.4369 + 0.0358S)q + 0.4903 + 0.0517S, 2 \le q \le 4 \\ (0.0005 + 0.000005S)q^2 + (-0.0128 - 0.0036S)q + 1.093 + 0.11S, 4 < q \le 15 \end{cases} \quad (1)$$

where q denotes the number of passengers; S the ratio of region length and width.

Patrons' travel time cost consists of three components: (i) the waiting time at home, C_W; (ii) the local tour travel time, C_T; (iii) the line-haul travel time, C_L; and (iv) the transfer time, C_R.

$$C_W = \sum_{n=1}^{N} \sum_{m=1}^{M} \alpha \cdot \left\{ \frac{1}{H_{p(m,n)}} \cdot E\left[Q_{p(m,n)} \cdot \frac{H_{p(m,n)}}{2}\right] + \frac{1}{2H_{p(m,n)}} \cdot E\left[Q_{p(m,n)} \cdot \left(\frac{1}{v_I}\left[(a + dS)\left(Q_{p(m,n)} + 1\right)^2 + (b + eS)\left(Q_{p(m,n)} + 1\right) + c + fS\right] \sqrt{\left(Q_{p(m,n)} + 1\right)lw + Q_{p(m,n)}\tau_p}\right)\right]\right\} \quad (2)$$

$$C_T = \sum_{n=1}^{N} \sum_{m=1}^{M} \frac{1}{2} \cdot \frac{1}{H_{p(m,n)}} \cdot E\left[Q_{p(m,n)} \cdot \left(\frac{1}{v_I}\left[(a + dS)\left(Q_{p(m,n)} + 1\right)^2 + (b + eS)\left(Q_{p(m,n)} + 1\right) + c + fS\right] \sqrt{\left(Q_{p(m,n)} + 1\right)lw} + Q_{p(m,n)}\tau_p\right)\right] + \sum_{n=1}^{N} \sum_{m=1}^{M} \frac{1}{H_{d(m,n)}} \cdot E\left[Q_{d(m,n)} \cdot \left(\frac{1}{2v_I}\left[(a + dS)\left(Q_{d(m,n)} + 1\right)^2 + (b + eS)\left(Q_{d(m,n)} + 1\right) + c + fS\right] \sqrt{\left(Q_{d(m,n)} + 1\right)lw} + \frac{Q_{d(m,n)}\tau_d}{2}\right)\right] \quad (3)$$

$$C_L = \sum_{n=1}^{N} \sum_{m=1}^{M} \frac{1}{H_{p(m,n)}} \cdot E\left[Q_{p(m,n)} \cdot \frac{(m-1)w + (n-1)l}{v_I}\right] + \sum_{n=1}^{N} \sum_{m=1}^{M} \frac{1}{H_{d(m,n)}} \cdot E\left[Q_{d(m,n)} \cdot \frac{(m-1)w + (n-1)l}{v_I}\right] \quad (4)$$

$$C_R = \sum_{n=1}^{N} \sum_{m=1}^{M} \frac{1}{H_{p(m,n)}} \cdot E\left[Q_{p(m,n)} \cdot \left(\frac{\tau_a Q_{p(m,n)}}{2} + t_{f-t} + \frac{H_t}{2}\right)\right] + \sum_{n=1}^{N} \sum_{m=1}^{M} \frac{1}{H_{d(m,n)}} \cdot E\left[Q_{d(m,n)} \cdot \left(\frac{\tau_b Q_{d(m,n)}}{2} + t_{t-f} + \frac{(k-1)H_{d(m,n)}}{2k}\right)\right] \quad (5)$$

where $E[\cdot]$ denotes the expected value; $Q_{p(m,n)}$ the random number of patrons carried by a bus serving subregion (m,n); v_I the bus cruising speed; a,b,c,d,e,f denote the parameters in Eq. (1); τ_p and τ_d the dwell time per stop in the collection and distribution direction, respectively; α the discount factor of the time value of waiting time at home; H_t the trunk-line headway; t_{f-t} and t_{f-t} the transfer delay per patron; k is an integer.

The agency costs include the distance-based operating cost, C_{vk}; and the time-based operating cost, C_{vh}.

$$C_{vk}=\sum_{n=1}^{N}\sum_{m=1}^{M}\left(\frac{\pi_v}{\theta H_{p(m,n)}}\cdot\left(\left((m-1)w+(n-1)l\right)+E\left[\left((a+dS)\left(Q_{p(m,n)}+1\right)^2+(b+eS)\left(Q_{p(m,n)}+1\right)+c+fS\right)\sqrt{\left(Q_{p(m,n)}+1\right)lw}\right]\right)+\frac{\pi_v}{\theta H_{d(m,n)}}\cdot\left(\left((m-1)w+(n-1)l\right)+E\left[\left((a+dS)\left(Q_{d(m,n)}+1\right)^2+(b+eS)\left(Q_{d(m,n)}+1\right)+c+fS\right)\sqrt{\left(Q_{d(m,n)}+1\right)lw}\right]\right)\right) \quad (6)$$

$$C_{vh}=\sum_{n=1}^{N}\sum_{m=1}^{M}\left(\frac{\pi_m}{\theta H_{p(m,n)}}\cdot\left(\frac{(m-1)w+(n-1)l}{v_I}+E\left[\frac{1}{v_I}\left((a+dS)\left(Q_{p(m,n)}+1\right)^2+(b+eS)\left(Q_{p(m,n)}+1\right)+c+fS\right)\sqrt{\left(Q_{p(m,n)}+1\right)lw}+\tau_p Q_{p(m,n)}\right]\right)+\frac{\pi_m}{\theta H_{d(m,n)}}\cdot\left(\frac{(m-1)w+(n-1)l}{v_I}+E\left[\frac{1}{v_I}\left((a+dS)\left(Q_{d(m,n)}+1\right)^2+(b+eS)\left(Q_{d(m,n)}+1\right)+c+fS\right)\sqrt{\left(Q_{d(m,n)}+1\right)lw}+\tau_d Q_{d(m,n)}\right]\right)\right) \quad (7)$$

2.3 Semi-flexible routing strategy model

The formulations of line-haul travel time and transfer time are the same as fully-flexible routing strategy. Thus, we listed the waiting time at home, the local tour travel time and agency costs.

$$C_W=\sum_{n=1}^{N}\sum_{m=1}^{M}\frac{\alpha}{H_{p(m,n)}}E\left[Q_{p(m,n)}\right]\left(\frac{H_{p(m,n)}}{2}+\frac{w_0}{3v_I}\right) \quad (8)$$

$$C_{Tp}=\sum_{n=1}^{N}\sum_{m=1}^{M}\frac{1}{2H_{p(m,n)}}E\left[Q_{p(m,n)}\left(\frac{1}{v_I}\left(\frac{Q_{p(m,n)}w_0}{3}+\frac{lw}{w_0}\right)+Q_{p(m,n)}\tau_p\right)\right]+\sum_{n=1}^{N}\sum_{m=1}^{M}\frac{1}{2H_{d(m,n)}}E\left[Q_{d(m,n)}\left(\frac{1}{v_I}\left(\frac{Q_{d(m,n)}w_0}{3}+\frac{lw}{w_0}\right)+Q_{d(m,n)}\tau_d\right)\right] \quad (9)$$

$$C_{vk}=\sum_{n=1}^{N}\sum_{m=1}^{M}\frac{\pi_v}{\theta}\left(\frac{1}{H_{p(m,n)}}E\left[\frac{Q_{p(m,n)}w_0}{3}+\frac{lw}{w_0}+(m-1)w+(n-1)l\right]+\frac{1}{H_{d(m,n)}}E\left[\frac{Q_{d(m,n)}w_0}{3}+\frac{lw}{w_0}+(m-1)w+(n-1)l\right]\right) \quad (10)$$

$$C_{vh}=\sum_{n=1}^{N}\sum_{m=1}^{M}\frac{\pi_m}{\theta}\left(\frac{1}{H_{p(m,n)}}\left(E\left[\frac{1}{v_I}\left(\frac{Q_{p(m,n)}w_0}{3}+\frac{lw}{w_0}+(m-1)w+(n-1)l\right)\right]+E\left[Q_{p(m,n)}\right]\tau_{p(m,n)}\right)+\frac{1}{H_{d(m,n)}}\left(E\left[\frac{1}{v_I}\left(\frac{Q_{d(m,n)}w_0}{3}+\frac{lw}{w_0}+(m-1)w+(n-1)l\right)\right]+E\left[Q_{d(m,n)}\right]\tau_{d(m,n)}\right)\right) \quad (11)$$

2.4 Optimization model

The generalized cost can be written as follows:

$$GC=C_W+C_{Tp}+C_{Td}+C_{Lp}+C_{Ld}+C_{Rp}+C_{Rd}+C_{vk}+C_{vh} \quad (12)$$

The optimization problem is formulated as follows:

$$GC = C_W + C_{Tp} + C_{Td} + C_{Lp} + C_{Ld} + C_{Rp} + C_{Rd} + C_{vk} + C_{vh} \tag{13a}$$

$\min GC$

subject to:

$$\lambda_p H_{p(m,n)} lw + 2\left(\lambda_p H_{p(m,n)} lw\right)^{\frac{1}{2}} \le K \tag{13b}$$

$$\lambda_d H_{d(m,n)} lw + 2\left(\lambda_d H_{d(m,n)} lw\right)^{\frac{1}{2}} \le K \tag{13c}$$

$$H_{d(m,n)} = kH_t \tag{13d}$$

$$H_{min} \le H_{p(m,n)} \le H_{max} \tag{13e}$$

$$\max\{H_{min}, H_t\} \le H_{d(m,n)} \le H_{max} \tag{13f}$$

$$M, N, k \in \{1,2,\dots\} \tag{13g}$$

$$w_0 \le \min\{l, w\} \tag{13h}$$

$$w_0 \in \left\{l, w, \frac{l}{2}, \frac{w}{2}, \frac{l}{3}, \frac{w}{3}, \frac{l}{4}, \frac{w}{4}, \dots\right\} \tag{13i}$$

where K denotes a feeder bus's patron-carrying capacity; H_{min}, H_{max} the minimum and maximum headways, respectively. Constraints (13h) and (13i) are for semi-flexible routing.

3 NUMERICAL CASE STUDIES

3.1 Benefits of accurate modelling

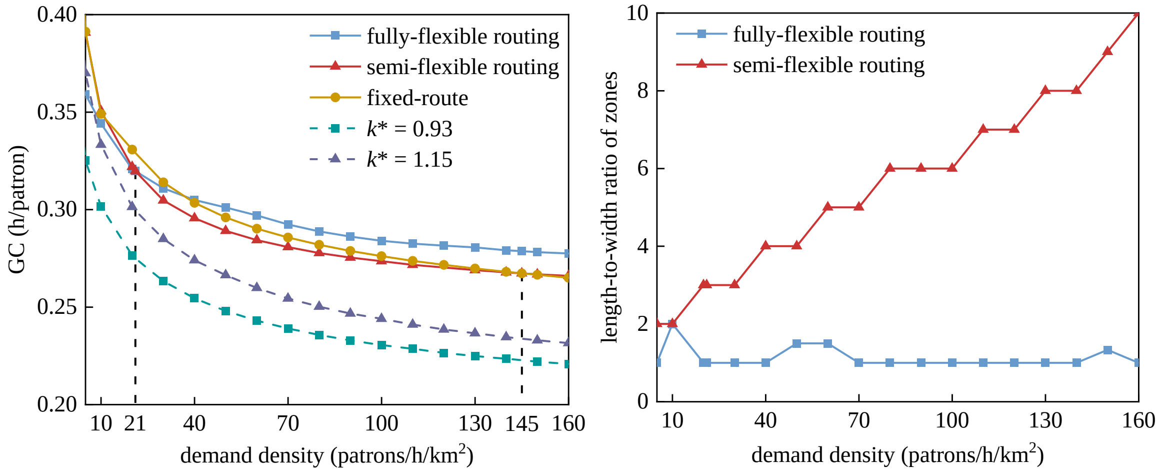
Our study improves on previous ones by using better models for estimating the lengths of tours and by including the effects of unpredictable demand in our designs. We compared our methods with older ones over 32 different scenarios, looking at how these changes affected costs and errors in estimating tour lengths and passenger time losses. The results reveal that for the fully-flexible routing strategy, the total cost errors can reach over 10%, sometimes nearly 19%. Even with a semi-flexible routing strategy, errors averaged above 6%.

Table 1 – *Percentage errors stemming from imprecise tour length estimation and omission of second-order stochastic effect*

Strategy	Fully-flexible routing				Semi-flexible routing	
Tour length estimation method	$k^* = \left(1.1055 - 0.008q + 1.0297\frac{S}{q}\right)$ (Yang et al., 2020)		$k^* = 0.93$ (Chakraborti and Chakrabarti, 2000)		$k^* = 1.15$ (e.g., Kim and Schonfeld, 1991)	
	Average	Maximum	Average	Maximum	Average	Maximum
Errors in GC	10.74%	18.64%	10.52%	12.15%	6.02%	11.84%
Errors in tour length	24.38%	49.46%	15.24%	35.91%	20.17%	30.93%
Errors in cumulative pick-up and drop-off time loss	12.54%	35.32%	12.67%	25.00%	19.24%	35.52%

3.2 Comparison of fully-flexible, semi-flexible and fixed-route services

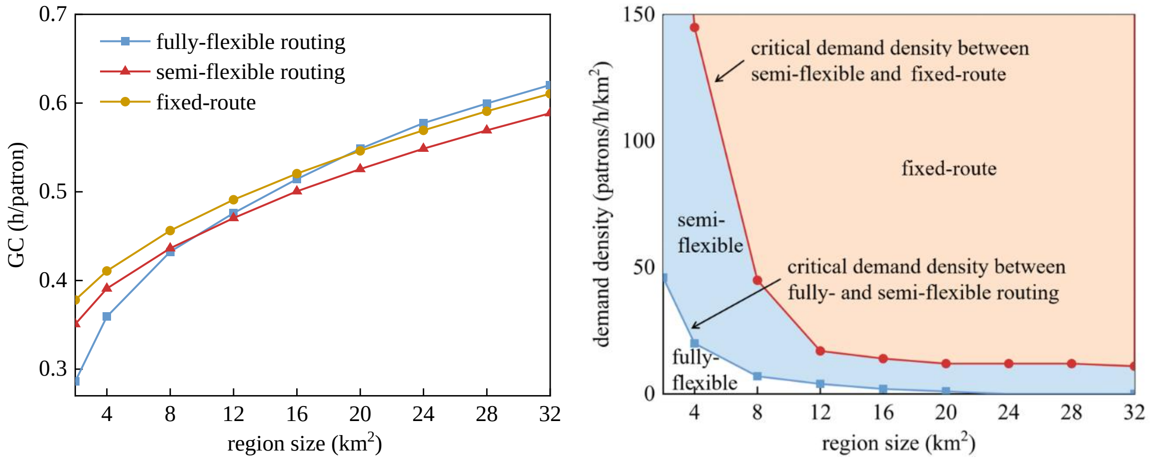
Firstly, our research compares fully-flexible, semi-flexible and fixed-route services across varying demand densities. Fully-flexible save more costs in less crowded areas, but semi-flexible services become beneficial as more demand densities. When demand densities are very high, fixed-route are the most cost-effective. Additionally, Fig. 3(b) shows that fully-flexible services are better for square-like areas, while semi-flexible services suit elongated areas.



(a) Generalized cost (b) Length-to-width ratio of zones

Figure 3 – *Effects of the demand density*

Then, we showed how bus strategies work for varies region sizes. The results indicate that for small areas, fully-flexible is best; for medium ones, semi-flexible is better; and for large areas, fixed-route is most efficient. From the variation of critical demand densities, fixed-route services benefit the most for larger regions and demand densities, followed by semi-flexible and then fully-flexible services. Increased flexibility in services tends to decrease the benefits gained from economies of scale.



(a) Generalized costs (b) Critical demand densities

Figure 4 – *Effects of the region size*

4 CONCLUSIONS

We developed analytical models for two DRC services: fully-flexible and semi-flexible routing strategies. We found that our models are more accurate in predicting demand stochasticity and tour length, improving fully-flexible strategy's accuracy by 10% and semi-flexible strategy's accuracy by 6%. We identified critical demand densities at the transition points between fully-flexible routing and semi-flexible routing, as well as between semi-flexible routing and fixed-route services. Fully-flexible routing is ideal for small, square areas and low demand, suitable for last-mile services. Semi-flexible routing works better for larger, rectangular areas with higher demand, like transport to airports or train stations.

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