Empirical Verification that Traffic Flow is on the KPZ Universality Class: Implications for Traffic Congestion

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Extended abstract submitted for presentation at the Conference in Emerging Technologies in Transportation Systems (TRC-30)
September 02-03, 2024, Crete, Greece

April 30, 2024

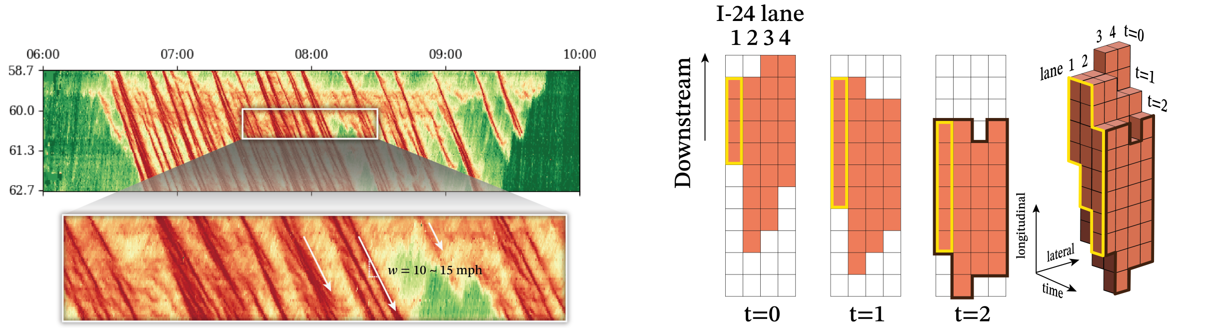
Keywords: Traffic Congestion, KPZ Universality Class, Fractal theory, Complex System, I-24 Motion

1 INTRODUCTION

Recent studies have significantly advanced our understanding of the fractal nature of traffic flow, with theoretical contributions from Laval (2023) and Laval (2024). These studies suggest that traffic flow is on the Kardar-Parisi-Zhang (KPZ) universality class. Specifically, traffic dynamic can be explained by KPZ exponents: α for traffic jam size distribution (the spatial roughness scaling), β for the time scaling of traffic fluctuations (the temporal growth scaling), and z for the correlation between total delay and lane-mile length (the space scaling to the time scaling). Laval (2023) indicates that traffic jam sizes adhere to a power-law with an exponent of 1/2 and that congestion dynamics demonstrate typical critical phenomena scaling, with z identified at 3/2, as $\alpha + z = 2$. Additionally, Laval (2024) finds that traffic fluctuations scale with time with an exponent $\beta = 1/3$, reflecting the growth exponent of KPZ, which describes how the interface width scales over time.

Building on these theoretical underpinnings, this study aims to empirically validate these critical scaling laws of 1+1 dimension KPZ universalities using real-world traffic data from the I-24 Mobility Technology Interstate Observation Network (MOTION). This dataset consists of observations along a 4.2-mile stretch of the I-24 freeway in Tennessee, USA, capturing both eastbound and westbound traffic across all four lanes (Gloudemans *et al.*, 2023). This dataset is unparalleled in its breadth and depth, capturing essential traffic congestion properties like post-accident discharge phenomena and consistent shock waves. This highlights the self-similarity of traffic congestion (Figure 1a), as seen when zooming into the time-space diagram still gives the same pattern.

By comparing empirical observations with theoretical scaling behaviors, this paper seeks to confirm the relevance of the KPZ class to traffic flow. We open the door to potential congestion mitigation strategies that could control KPZ scaling exponents to maintain the traffic system in a sub-critical state. Not we can mitigate congestion effectively until proposing novel control methods to keep the network in a sub-critical state by understanding its complex dynamics.



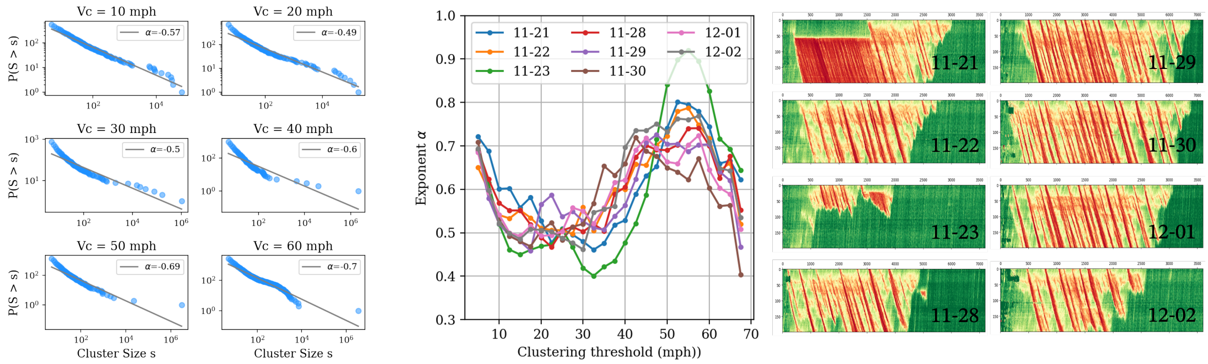
(a) (b)

Figure 1 – *Visualizations of traffic dynamics: (a) Time-space diagram showing self-similarity on consistent shock wave speed and (b) 3D traffic jam clustering.*

2 Scaling of Traffic Jam Sizes: KPZ Exponent $\alpha = 1/2$

We begin by verifying the KPZ exponent α, known as the roughness exponent, through estimating the power-law exponent of the traffic jam cluster distribution, which follows $P(S > s) \sim s^{-\alpha}$. To calculate the traffic jam cluster, we reconstruct the speed field from trajectory data within a defined small space and time range ($dx = 0.02$ mile, $dt = 2$ seconds), following Edie's generalized definition. Missing data points are imputed using an adaptive smoothing method, as implemented by Ji *et al.* (2024); however, this smoothing is selectively applied only to missing points in the data, preserving original data integrity where complete.

We apply a speed threshold to cluster data points where speed falls below. Inspired by Zhang *et al.* (2019), we create 3D clusters by stacking time-space diagrams for each lane without averaging them (Figure 1b). This preserves lane-specific details and enhances the granularity and precision of our analysis. Note that this 3D clustering is not necessarily the typical 2D spatial clustering, our approach adapts to highway conditions where vehicles are restricted to lane changes and cannot change their direction, still making it 1+1 dimension for the KPZ class.



(a) (b)

Figure 2 – *Detailed analysis of α: (a) Log-log survival plot of jam size distribution (Nov. 28th); (b) Estimation of α by varying thresholds (time-space diagram for each day as reference)*

Our analysis spans eight days of significant westbound congestion. Given the challenges of defining a precise threshold for the critical state in real-world traffic, we experimented with various thresholds to estimate the power-law exponent, as illustrated in Figure 2. We observed that the traffic jam size distribution exhibits a power-law behavior. Importantly, the trend of exponent changes remained consistent across eight distinct dates, while having different congestion patterns, with the exponent consistently approximating 0.5 at thresholds ranging from 15 to 35 mph, thereby corroborating the findings of Laval (2023, 2024). Unlike the findings of Zhang *et al.* (2019), Zeng *et al.* (2019), who reported a cluster size exponent of approximately 2.3 based on the number of links below the speed threshold, our results reveal a notable sensitivity to the chosen speed threshold. This prompts further discussion on how the cluster size distribution exponent might vary with urban versus highway settings or depending on the granularity of cluster definition.

3 Traffic Fluctuation grows with time: KPZ Exponent $\beta = 1/3$

The KPZ exponent $\beta = 1/3$ describes how the standard deviation of traffic density, $\sigma(t)$, evolves over time, suggesting that fluctuations in traffic density grow proportionally to $t^{1/3}$. Unlike Brownian motion, where $\sigma(t) \sim t^{1/2}$ indicates a lack of memory and complete randomness, the exponent of 1/3 suggests a subdiffusive process where fluctuations spread slower than in a purely random system. Notably, the Hurst exponent, which measures self-similarity or long-range dependence in time series, aligns with the KPZ β when the time series is a density fluctuation, explicitly demonstrating the equivalence.

To analyze the temporal scaling behaviors, we measure traffic density through the instantaneous vehicle count per second and calculate the Hurst exponent. Figure 3 presents the density time series over four days, alongside Hurst exponents calculated for 10-minute and 90-minute windows with a 10-minute step size. The legend at the top indicates the Hurst exponent calculated across the entire four-hour span for each date. Notably, the Hurst exponent shows considerable variability with a 10-minute window size, averaging around 0.39±0.037. As the window size expands to 90 minutes, the Hurst exponent stabilizes at approximately 0.31±0.020. While this analysis includes only four days for conciseness, a similar trend is observed across all eight days studied. Although these results do not conclusively establish the Hurst exponent as consistently being 1/3 in traffic flows, they indicate a tendency towards KPZ-type scaling. This pattern suggests that traffic flow disturbances do not propagate entirely at random, hinting at underlying correlations within the traffic system. Further investigations into these temporal growth patterns in critical states are necessary to better understand the dynamics at play.

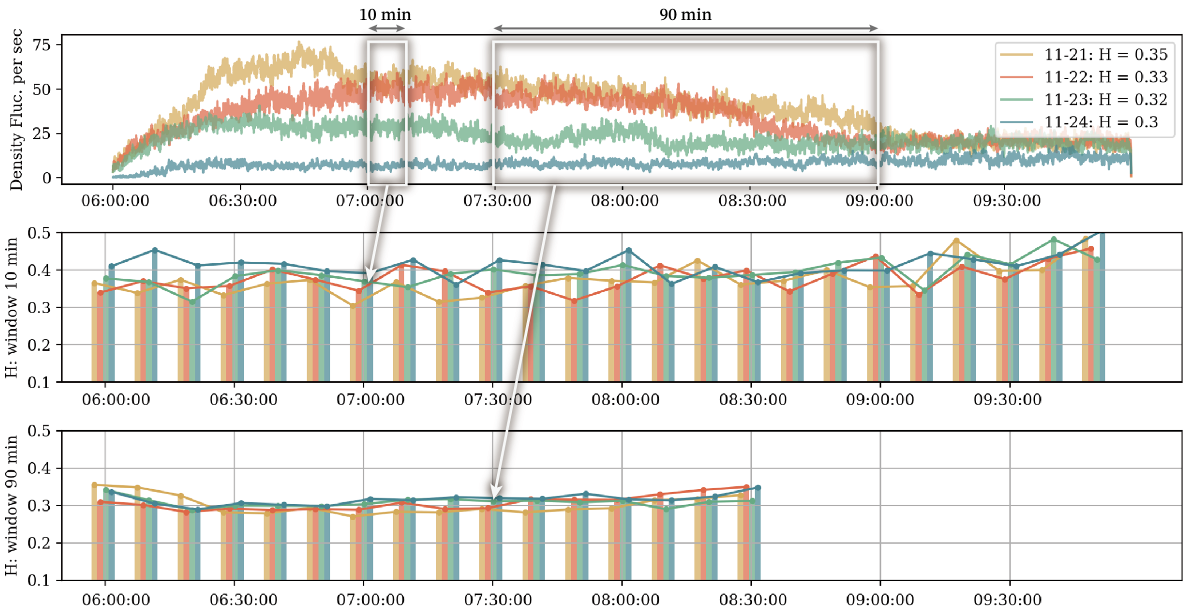
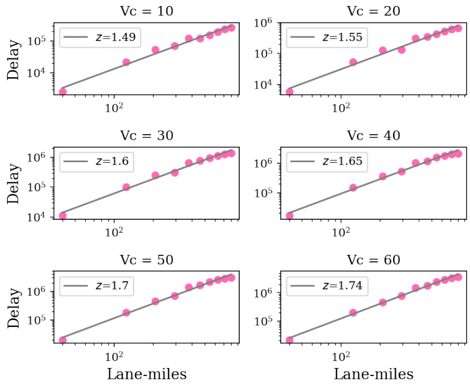


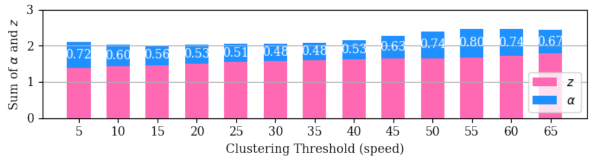
Figure 3 – *Density fluctuation time series for four days (Top). Computed Hurst exponents using 10-minute (Middle) and 90-minute time windows (Bottom)*

4 Scaling of space to time: KPZ Exponent $z = 3/2$

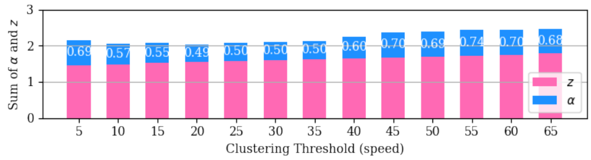
The dynamical exponent, $z = 3/2$, indicates total delays, Δ, scale with the total lane-miles of the network, L_{net}, denoted by $\Delta \sim L_{\text{net}}^{3/2}$. This scaling law implies how congestion-induced delays grow with the size of the system. The total delay can simply be calculated by the sum of all traffic jam sizes. We analyze this relationship by dynamically varying and shifting the KPZ window to highly congested areas, defined by $(t, l) = (c\ell^{3/2}, \ell)$, where t and ℓ represent the time and space intervals of the window, respectively. This analysis reveals a scaling relationship between these two variables, as illustrated in Figure 4a. Figure 4b and 4c display the sum of α and z for various thresholds; a sum close to 2 around the critical speed confirms the KPZ universality class's applicability to urban traffic congestion phenomena.



(a) *Nov. 28th*



(b) *Nov. 21st*



(c) *Nov. 28th*

Figure 4 – *Detailed analysis of z: (a) Log-log plot of delay and lane-miles; (b)&(c) Sum of α and z by different thresholds. Used same α from Figure 2.*

5 Conclusion

This study has demonstrated the existence of KPZ universality class exponents—α, β, and z—in empirical traffic data, having values of $1/2$, $1/3$, and $3/2$, respectively. While our findings closely approximate theoretical expectations, further statistical verification is essential. Future research will focus on: (1) analyzing data only on critical states, (2) addressing other exponents related to the largest cluster size and cluster lifetime, and (3) developing traffic control strategies by identifying methods that can keep KPZ scaling exponents below critical thresholds.

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