A two-stage stochastic optimization model for sidewalk robot food delivery systems

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1 INTRODUCTION

The rapidly expanding market of the online food delivery (OFD) industry poses a significant challenge to last-mile delivery operations. Currently, companies mostly rely on on-demand workers to offer last-mile services. This poses multiple challenges such as safety concerns, payment fairness, delivery efficiencies, etc. Emerging technologies offer new alternatives for providing OFD services, and a sidewalk delivery robot (SDR) system is one such option. Figure 1(a) is an example of the SDR. With the development of autonomous driving technologies, box-sized robots navigating through sidewalks for food and grocery deliveries have become real-world applications. Analytical tools are needed to optimize sidewalk robot online food delivery (SROFD) systems at scale. An SROFD operator needs to determine the optimal resources required to properly serve an area with uncertain demands. Such decisions include charging infrastructure allocation to different depots and robot fleet sizing.

To the best of our knowledge, no prior study applies the resource allocation problems with demand uncertainty on an SROFD system, which has its unique characteristics. The SFORD system's operation closely follows the Pickup and Drop-off Problem with Time Windows (PDPTW), and the pickup and drop-off stops are directly linked. In addition, SDRs have limited battery constraints that often require charging activities during operations.

Based on these features, we propose a two-stage stochastic problem that optimizes the resource allocation strategy of a multi-depot SROFD system. Each depot has its respective SDR fleet serving a dedicated area. A subset of depots also has battery swapping facilities, similar to the one illustrated in Fig. 1(b), which has a fixed time for battery swapping and replenishes batteries to full. During an operation period, only one battery swapping activity is allowed for each SDR. The first stage problem determines the optimal fleet size for each depot and which stations have battery-swapping capabilities. The chosen depots are assumed to have unlimited battery supply. The allocation of the battery swapping stations to the depots impacts the routing patterns of the robots. The second stage solves the routing problem with a late arrival penalty based on the first stage solution.

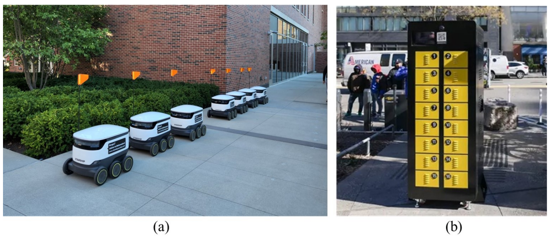

Figure 1 – *An example of (a) SDRs, (b) a battery swapping facility (Ginsburg et al. (2024))*

The two-stage stochastic programming problem is solved using an original solution algorithm that uses continuous approximation (CA) in the first stage to quickly obtain a reduced solution space from the first stage problem. For the second stage, we propose a heuristic algorithm that provides high quality solutions under a short running time based on the first stage solution space. The second stage results can be used to further guide and adjust the CA model used in the first stage model. Finally, the sample average approximation (SAA) methods is applied and we choose the solution with the lowest average second stage objective value as the final result. Fig. 2 illustrates the structure of the two-stage model.

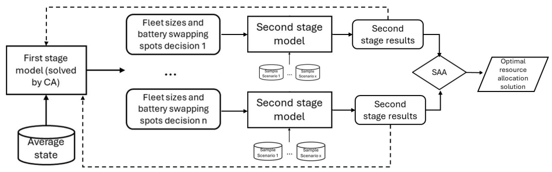

Figure 2 – *Illustration of the model structure*

2 METHODOLOGY

Each depot $z \in Z$ has its respective service area that shares the same label. A set of pickup locations and drop-off locations are uniformly distributed across the area. Pickup and drop-off locations are paired up for order generation. In a defined operation period, random orders are generated on each order pair. Orders are generated based on each time step, and each pair generates orders following Poisson distribution with a respective mean value. Each order is assigned with a unique label π. Earliest pickup time and the due time for drop-off are assigned for each order based on the order generation time. Each depot is both the start and the end node of routes in its area. Each area has a dedicated battery swapping node that has the same location as the closest depot with battery swapping capacity. The first stage decision variables are the fleet size v_z in each depot z and the battery swapping facility allocation decision w_z. Given $v_z \in V$ and $w_z \in W$, we define the function $C(\omega, V, W)$ as the minimum operational cost for scenario ω. The cost of each SDR is α and the battery swapping installation cost is θ per depot. The first stage model has an objective written as Eq.(1), along with standard non-negativity and budget constraints (not shown in this abstract).

$$min \sum_{z} (\alpha v_z + \theta w_z) + E[C(\omega, V, W)] \tag{1}$$

For each scenario ω, the second stage model minimizes the operation cost. $C(\omega, V, W)$ includes the total routing distance and the penalty cost applied to the number of orders with late returns to their depot. It is calculated with the realized order demand given V and W. $x^z_{i,j,k} \in \{0, 1\}$ is a decision variable to route the SDR $k \in 1, ..., v_z$ from node i to node j for each depot z. The unit cost per distance is β. $h^z_{i,j}$ is the distance between the two nodes i, j based on the their locations. An order drop-off due time l^z_π is used to penalize late deliveries. A binary variable r^z_π is introduced for each order π in area z, and it is set to 1 if late. The penalty cost per order violation is a constant value γ. The second stage objective function is written as Eq.(2).

$$C(\omega, V, W) = min\ \beta \sum_{z,i,j,k} h^z_{i,j} x^z_{i,j,k} + \gamma \sum_{z,\pi} r^z_\pi \tag{2}$$

The constraints of the second stage model follow the classic electric vehicle routing problem with time window (E-VRPTW) structure proposed by Schneider *et al.* (2014) with several major adjustments. First, the model is modified to comply with a fleet of identical SDRs. Second, constraints are added to ensure the visited nodes follow the pickup-then-drop-off rule for each order. Since one of the model's objectives is to minimize the number of late orders, additional constraints are introduced to track the late status of each order while dropping the hard time window constraint. Finally, constraints are adjusted to adhere to the rule that a battery swapping activity is limited to at most once for each SDR tour. If a SDR's base depot does not have a battery swapping facility, it will travel to the closest depot with available battery swapping facilities before returning to its assigned depot. The SDR travel time and the energy consumption rate are calculated based on shortest distances.

Order demands are independently generated using Poisson distribution for each order pair during each time step. Therefore, the expected number of orders in a fixed period is the simple summary of the mean values. With the locations being uniformly distributed across the area, the continuous approximation (CA) method could be used to estimate the average total travel distance without knowing the operation details. A CA based model inspired by Figliozzi (2009) and Bergmann *et al.* (2020) is developed to quickly find a tight solution space for the first stage model. The model considers the added distance caused by time constraints, battery swapping activities, and order compliance requirements. It also considers the average number of late orders.

Multiple scenarios ω are generated to solve the second stage model given the first problem solutions. The average objective value among the scenarios is calculated to approximate $E[C(\omega, V, W)]$ for each V and W in the first stage solution space. Because the second stage model in each problem setting is NP-hard, we propose a solution method that uses heuristics to generate routing without charging, then inserting charging into the established routes to produce the final routing. An Adaptive Large Neighborhood (ALN) heuristic is used to generate route without charging, and a method modified from Froger *et al.* (2019) is used for charging insertion. The model is solved for each depot z independently and then added together.

3 NUMERICAL EXAMPLE

We use a toy example shown in Fig. 3 to illustrate the model in a bounded Euclidean space. Two depots A and B with their respective service areas A and B are involved. Pickup, drop-off, and depot locations are predetermined in the two areas. The two depot locations are also the candidate battery swapping locations. Combinations of all pickup and drop-off locations are grouped to form order pairs. A random number of orders is generated among the order pairs in two 15-minute intervals. For each order generated from an order pair during an interval, the earliest order pickup time is defined as the end time of the interval, and no penalty would occur if it is dropped off within an hour after the earliest pickup time. For example, if order pair 1-5 in depot area A during the first 15-minute interval generates an order, this order's earliest pickup time is 15 minutes after the start time at point 1, and the latest drop-off time is at the 75-minute mark at point 5.

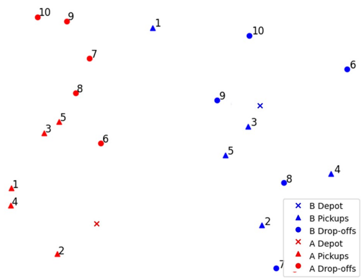

Figure 3 – *Illustration of the toy example*

The mean number of orders in one 15-minute interval is 3 orders. An expected number of 6 orders in each area is generated for the whole period. Each stop requires 2-minute service time, and the battery swapping operation also needs 2-minute to complete. The cost of one SDR is 2, the operation cost is 0.3 per unit distance, the late penalty of each order is 1, and the battery swapping installation cost is 3. The battery can cover up to 5 distance units and each SDR can hold up to 6 orders. The first stage model is to determine the optimal fleet size and battery swapping locations based on the expected number of orders. Table 1 summarizes the top five first stage solutions using the CA model and corresponding average second stage results using 10 sampled scenarios. The top two solutions in the first stage model align with the results from the second stage model and both solutions provide significant cost savings compared with others. It illustrates the effectiveness of using the CA model to significantly reduce the first stage solution space, which could significantly reduce the calculation time required for the second stage problem while pertaining the result accuracy.

For the full paper, randomized instances are generated to evaluate the computational efficiency and accuracy of the solution algorithm against benchmarks. Solutions directly solved by existing solvers will be used as benchmarks. In addition, a large instance with more than a hundred orders and multiple depots will be solved and presented to showcase the model's capability. By using the proposed approach, the stochastic model can be expanded with other factors involved such as battery swapping facility with battery capacity and adding battery capacity as part of the decision variables.

Table 1 – *Results of the toy example*

Battery Swapping Location A, B	0, 0	0, 1	1, 0	1, 0	0, 1
Fleet Size A, B	2, 2	1, 1	1, 1	1, 2	1, 2
First stage result (CA model)	11.29	11.29	12.81	13.84	12.29
Second stage result (10-scenario average)	11.42	11.87	12.83	12.88	12.93

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