Understanding Physics and AI Synergy in Car-following Models

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1 Introduction

This paper aims to understand how physics-based and AI models interact and supplement each other in modeling car-following (CF) behavior. Physics-based CF models are grounded in robust theoretical frameworks, offering a level of interpretability and trust in their output. Nonetheless, achieving a simplified theoretical exploration comes at the expense of modeling complex dynamics and interactions in generalized traffic environments. This makes adopting traditional CF models to automated vehicles (AVs) control more challenging, given the crucial need to realize generalizability. Conversely, the emerging AI models (i.e., deep learning (DL)) offer unique advantages in learning and adapting from data, particularly capturing non-linear complex relationships. Yet, this requires a vast amount of high-quality training data that is often limited or unavailable. More importantly, the AI methods lead to models with few to no interpretable insights, impairing traffic-level understanding.

Consequently, we see a shift in adopting AI-based models to supplement traditional CF models – where they might fail – giving rise to physics-informed artificial intelligence (AI) models, most famously physics-informed neural networks (PINN) (Mo *et al.*, 2021, Cuomo *et al.*, 2022). The rigorous analysis of their approximation efficiency remains unexplored. In this work, we want to understand the behavior of these physics-informed AI models. This entails analyzing how AI models interact with traditional CF models and what expectations we have on the synergy between the two.

Incorporating physics-informed AI into CF models brings multiple dimensions of analysis: predictive accuracy, data needs, convergence, interpretability, etc. All these play a role in physics-AI synergy. However, in this abstract we focus on the accuracy metric. Our interest here is to investigate how the underlying physical features in different CF models across different families synergies with AI models. We do so through theoretical error bound modeling and sensitivity-based simulations.

2 Analysis of Physics-AI Synergy

This section examines how different CF families interact with or support AI counterparts by analyzing the generalized error bounds (GEB) of CF behavior approximation. Fig. 1 illustrates a synergy training paradigm bridging physics and AI, distinct from both the physics-only estimation and AI-only estimation approaches. We denote a specified CF model m from family n as $F_{mn}(\cdot|\vec{\theta}_{mn})$, where, $\vec{\theta}_{mn}$ represents the parameters in F_{mn}. Likewise, we have $D_{mn}(\cdot|\vec{\alpha}_{mn})$ to represent different DL algorithms, where $\vec{\alpha}_{mn}$ represents admissible set of tuning parameters. A typical CF estimation using PINN framework is to train the optimal α^*_{mn} so to have $\vec{\theta}^*_{mn}$ for F_{mn}, as Eq. 1.

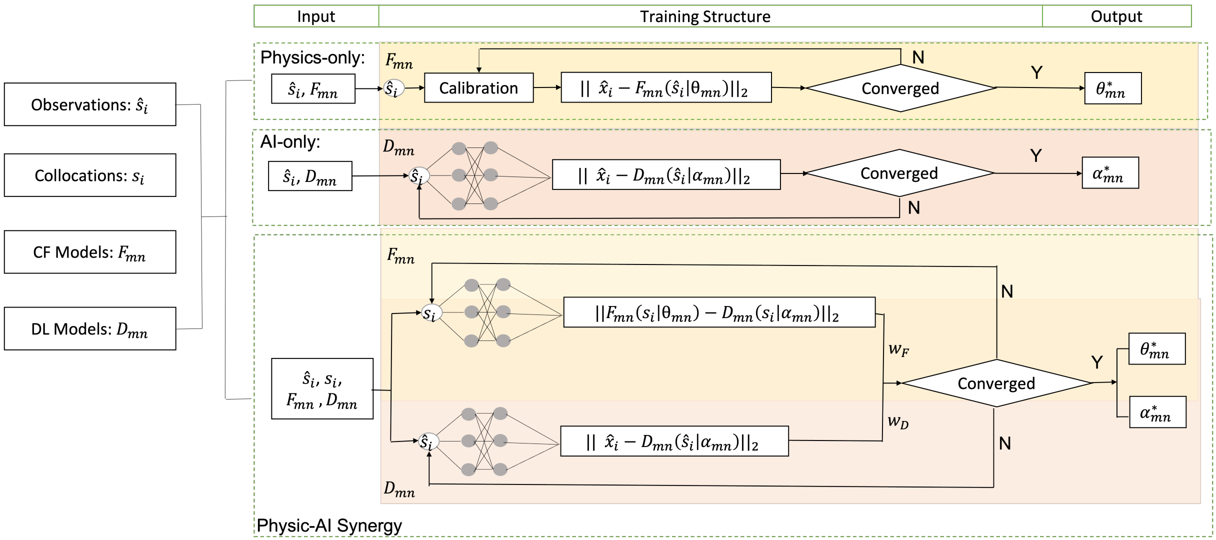


Figure 1 – *Physics-AI Synergy Training Paradigm*

$$\alpha^*_{mn} = \arg\min_{\alpha_{mn}} w_F(\|F_{mn}(s_i(t)|\vec{\theta}_{mn}) - D_{mn}(s_i(t)|\vec{\alpha}_{mn})\|_2) + w_D(\|\hat{x}_i(t) - D_{mn}(\hat{s}_i(t)|\vec{\alpha}_{mn})\|_2) \tag{1}$$

where w_F, w_D are the weights to balance the roles of F_{mn} and D_{mn}, $\hat{x}_i(t)$ is the observed trajectory for follower, i, at time step, t. $\hat{s}_i(t)$ is the observed state in the current time step (e.g., leader position, relative speed, etc.) for predicting the trajectory in the next time step, $s_i(t)$ is the collocation state to augment $\hat{s}_i$ with additional samples from the multi-dimensional distributions of $\hat{s}_i(t)$ learned by DL, all while adhering to the physics constraints.

With training sets for s_i comprising N_1 data points and for $\hat{s}_i$ with N_2 data points, we further apply the quadrature rule to compute the training errors, $\epsilon_1 = (\sum_{i=1}^{N_1} w_{s_i}|F_{mn}(s_i|\vec{\theta}^*_{mn}) - D_{mn}(s_i|\vec{\alpha}^*_{mn})|^2)^{\frac{1}{2}}$ and $\epsilon_2 = (\sum_{i=1}^{N_2} w_{\hat{s}_i}|\vec{\hat{x}_i} - D_{mn}(\hat{s}_i|\vec{\alpha}^*_{mn})|)^{\frac{1}{2}}$, where w_{s_i} and $w_{\hat{s}_i}$ are the quadrature weights related to the underlying order, d_w. We have the total training error under regularization $\epsilon_T = (\epsilon_1^2 + \lambda\epsilon_2^2)^{\frac{1}{2}}$, where λ is the regularization term (Mishra & Molinaro, 2022).

The generalized total error with noise term v, ϵ_G could be defined as, Eq. 2, with the conditional stability estimates for convergence (Mishra & Molinaro, 2022), ϵ_G could be bounded by Eq. 3 for an effective PINN.

$$\epsilon_G = \left\|F^*(\cdot|\theta^*) - F_{mn}(\cdot|\vec{\theta}^*_{mn})\right\|_2 \tag{2}$$

where $F^*(\cdot|\theta^*)$ is the ground-truth CF model.

$$\epsilon_G \le C_0\left[w_F\epsilon_1 + w_D\epsilon_2 + C_1^{\frac{1}{2}} N_1^{\frac{-\eta_1}{2}} + C_2^{\frac{1}{2}} N_2^{\frac{-\eta_2}{2}} + \|v\|_2\right] \tag{3}$$

where C_0, C_1, C_2 are bounded constants, dependent on the learning accuracy of $D_{mn}(\cdot|\alpha^*_{mn})$ and $F_{mn}(\cdot|\vec{\theta}^*_{mn})$. η_1 and η_2 are related to the regularity of the underlying integrand (i.e, data input space).

Leveraging Eq. 3[1], we can infer that (1) increasing the number of training points helps reduce the noise error, and (2) the most effective combination of physics and AI would likely be a well-trained DL model (i.e., with lowest error) and a well-defined CF model (i.e., with desired physical properties). This underscores the foundational assumption of CF models that DL algorithms can effectively enhance parameterization to better describe CF behavior.

3 Results

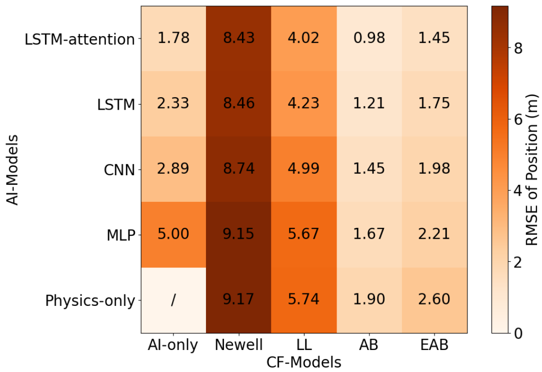
We selected the Newell model and its three extensions – Laval-Leclercq (L-L), the Asymmetric Behavioral Model (AB), and the Extended Asymmetric Behavioral Model (EAB) – as representative CF models for analysis. These models progressively incorporate additional parameters to better describe CF behavior. Correspondingly, we chose AI counterpart models with increasing model complexity (e.g., more representations to capture the spatial or temporal dependence) for better accuracy: Multilayer Perceptron (MLP), Convolutional Neural Network (CNN), Long Short-Term Memory (LTSM), and LTSM with an attention mechanism (LTSM-attention).

In our analysis we use a small dataset of 16 stop-and-go trajectories extracted from NGSIM-US101 dataset, partitioned into training and testing sets at a 3:1 ratio. Then $\hat{s}_i$ includes 12 trajectories with $N_2 = 7500$. We maintain a ratio of $\frac{N1}{N2} = 100$ to generate s_i using Latin hypercube sampling strategy (Raissi *et al.*, 2019). We assign equal weights of $W_F = W_D = 0.5$. Finally, our evaluation criteria is based on root mean square errors (RMSE) between the observed and simulated positions.

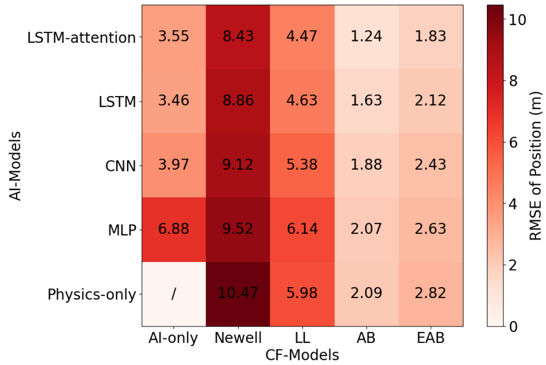
The followings are some major findings drawn from Fig. 2. (1) **The optimal physics-AI synergy is achieved by combining the best performing models from both physics-only and AI-only.** Interestingly, the complexity of a CF model – as indicated by its number of parameters – is less critical than its descriptive accuracy. The CF model that results in lowest RMSE error, will be a better choice for optimal physics-AI synergy. The training error results from the physics-only model (bottom row in Fig. 2a) indicate that the AB model outperformed others, whereas the DL-only training identified the LSTM-attention model as the most effective (leftmost column in Fig. 2a). These outcomes, together with Eq. 3, imply that a hybrid approach combining AB with LSTM-attention could provide the optimal solution, as evidenced by the synergy results (Fig. 2a). (2) **More elaborated AI model could help the CF model achieve better accuracy**, as evidenced by the decreasing RMSE from the bottom to the top in each column. More sophisticated AI models are adept at automatically selecting and extracting the most relevant features, aiding CF models in learning more accurate parameters. (3) **The effectiveness of the physic-AI synergy heavily depends on the performance of the chosen CF model**. Choosing a poor CF model may limit the improvement achieved through AI integration. A good CF model could highly enhance the DL generalizability. The synergy performance of the Newell model or LL model is inferior to that of the AB and EAB, and even falls short of the AI-only performance. This discrepancy arises because the AB and EAB models are designed to capture more heterogeneity in driving behavior through various reaction patterns, thereby guiding AI towards more precise learning. By analyzing training and testing errors of the AB/EAB-AI synergies we see a good ability for generalizability. (4) **The AI model could help complex physics-models in reducing overfitting.** This can be inferred through the observation that AB-AI model outperforms EAB-AI model. Note that, AB model is specifically tailored for human-driven vehicles (HDVs) under oscillation, whereas the EAB is designed for commercial AVs. While the EAB model is indeed more flexible than the AB model, it can suffer from overfitting due to its higher number of parameters, which may not be necessary for HDVs.

[1]From Eq. 3, it is evident that the bound depends on the number of training points, N. This complicates the inference of error bounds for AI models characterized by memory loss (such as recurrent neural networks or various configurations), because part of the data from N is lost during the training process. We do not address this complexity in our current model, but plan to explore it in future extensions of this work.

Interestingly, there are exceptions where EAB+LSTM-attention performs better than AB+CNN and AB+MLP. This indicates that a well-structured DL could alleviate the overfitting issue to certain extent. These findings are further substantiated by their performance on the testing dataset (Fig. 2b).



(a) *Training*



(b) *Testing*

Figure 2 – *Physics-AI Synergy*

4 Discussion and Planned Work

The preliminary investigation done in this work, reveals important insights that require further attention. A pivotal finding from our research highlights the significance of training data in achieving optimal physics-AI synergy. One can clearly note that the performance of a physics-based CF model (or family) varies with the driving scenarios (or traffic properties) represented in the data. This variation stems from the inherent design of CF models to optimally represent specific aspects of driving behavior, such as stop-and-go, free flow, high-speed conditions, etc. Consequently, the most suitable CF model for achieving optimal physics-AI synergy is the one that, in expectation, exhibits the best performance across all driving scenarios. While our current investigation is confined to predictive accuracy of stop-and-go data, the full paper will explore how different driving scenarios influence the outcomes observed in this study. More performance metrics would be included to rigorously examine the physics-AI synergy.

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