Where do Vulnerable Road Users Go at Automated Unsignalized Intersections? An Analysis of Interactions

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1 INTRODUCTION

The explosion of the autonomous driving technology has raised expectations for the future mobility. Unsignalized intersections serve as a typical example, demonstrating the potential for traffic participants to pass through intersections in a manner that both conflict-free and efficient, without reliance on traffic signals (Qin *et al.*, 2024). In the context of automated unsignalized intersections, interactions between connected and automated vehicles (CAVs) have been extensively modeled. Nevertheless, the interaction between vulnerable road users (VRUs) (e.g., pedestrians, bicyclists, scooterists and wheelchair users) and CAVs at unsignalized intersections is considered a challenge since the presence of blind spots and the inherent vulnerability of VRUs to traffic crashes (Bautista-Montesano *et al.*, 2022, Qin *et al.*, 2024). This challenge prompts a crucial question: what future developments are needed to better integrate VRUs into the evolving intelligent transportation system, especially from the transportation equity perspective?

To address this issue, this paper introduces the differential game to explore the intricate and dynamic interactions between VRUs and CAVs at unsignalized intersections. Through rigorous analysis of their potential conflicts, this study further delineates a management strategy tailored for this mixed traffic flow. The primary contributions of the paper include:

- It offers a dynamic interaction-based cooperative decision-making approach for heterogeneous traffic participants (i.e., VRUs and CAVs) (DI-CDM) at unsignalized intersection to generate a safe and efficient strategy.
- The analysis focuses on multi-player, continuous, and dynamic interactions with the goal of deriving the Nash Equilibrium, which represents a fair and conflict-free trajectory strategy.

2 METHODOLOGY

Heterogeneous traffic participants have different motion patterns and preferences, and the complexity of analyzing their behavior is heightened by the multi-player, continuous, and dynamic interactions. In this study, we focus on three predominant categories of traffic participants: pedestrians, bicyclists, and CAVs, without restricting the generality of our findings. As described by Qin *et al.* (2024), a typical four-leg unsignalized intersection features thirty-two (32) conflicts points among motorized traffic and twenty-four (24) between non-motorized and motorized traffic. These points highlight areas where interactions between different traffic participants are most likely to occur. To provide an in-depth description of the dynamics and interactions of traffic participants, this section includes (i) the modeling of the motion patterns of traffic participants, which aims to accurately capture their behaviors and movements within the intersection, and (ii) the analysis of their interactions through the lens of differential game.

2.1 Modeling the motion of traffic participants

For the purpose of simplicity and considering that vehicles typically operate at low speeds within intersections, this study introduces the vehicle kinematic model, known as the bicycle model, to simulate the behavior of CAVs.

$$\frac{dx(t)}{dt} = v(t)cos(\psi(t)); \ \frac{dy(t)}{dt} = v(t)sin(\psi(t)); \ \frac{d\psi(t)}{dt} = \frac{v(t)}{l_f + l_r}tan(\delta(t)) \tag{1}$$

where l_f and l_r are the distance between the center of mass and the front and rear axle, respectively. $\delta(t)$ is the front wheel angle. The coordinates of the vehicle's rear axle axis, positioned in the inertial coordinate system OXY, are expressed as $(x(t), y(t))$. $v(t)$ indicates the vehicle velocity at time t and $\psi(t)$ is the yaw angle at time t. The kinematic model applied to bicycles is identical to that of CAVs, with distinctions arising solely from the parameters involved. In modeling pedestrian dynamics, a straightforward approach in Hoogendoorn & HL Bovy (2003) is adopted.

$$\frac{dl(t)}{dt} = v(t) \tag{2}$$

where $l(t)$ is the position of the pedestrian, and $v(t)$ is his or her velocity at time t. For clarity in distinguishing among the entities, the subscripts c, b, and p will be employed hereafter to denote CAVs, bicyclists and pedestrians, respectively.

2.2 Analyzing interactions using the differential game

Differential game is notably effective for modeling the behaviors of traffic participants, which is capable of capturing the dynamic interactions among pedestrians (Hoogendoorn & HL Bovy, 2003), bicyclists (Hoogendoorn *et al.*, 2021) and CAVs (Hang *et al.*, 2022). Given the excellent performance of the differential game in traffic flows with a single type of traffic participant, in this paper, we try to introduce it into a mixed traffic flow consisting of VRUs and CAVs.

Considering a differential game over a continuous time period, i.e., $t \in [t_0, t_0 + T]$, where t_0 marks the game start time and T is the game duration. The state of the game is encapsulated by a vector $g(t) = (g_c^1(t), g_c^2(t), \cdots, g_c^n(t), g_b^1(t), g_b^2(t), \cdots, g_b^m(t), g_p^1(t), g_p^2(t), \cdots, g_p^q(t)) \in G$, where $G \subseteq \mathbb{R}^{n+m+q}$ denotes the set containing all possible states. n, m and q also denote the number of CAVs, bicyclists and pedestrians, respectively, engaged in the game. Furthermore, the state transition of the control system can be delineated through an ordinary differential equation, complemented by specific initial conditions:

$$\dot{g(t)} = f(t, g(t), u(t)); \ g(0) = g_0 \tag{3}$$

where $u(t) = (u_c^1(t), u_c^2(t), \cdots, u_c^n(t), u_b^1(t), u_b^2(t), \cdots, u_b^m(t), u_p^1(t), u_p^2(t), \cdots, u_p^q(t))$ is the vector of decision variables at time t. We employ $u(t) \in U(x(t), t)$ to represent the constraints on the decision variables, which include upper and lower bounds of the speed and the front wheel angle.

Within the framework of the differential game, it is posited that the primary objective of each traffic participant is to maximize the personal benefit (J).

$$J = \int_{t_0}^{t_0+T} e^{-\rho t} L(t, x(t), u(t)) dt + e^{-\rho(t_0+T)} \Phi(t_0 + T, x(t_0 + T)) \tag{4}$$

where $L(t, x(t), u(t))$ is the running cost over time interval $[t, t + dt]$, ρ is a discount factor, and $\Phi(t_0 + T, x(t_0 + T))$ represents the cost of the terminal state.

To ensure safety, we utilize the estimated time difference between any two traffic participants approaching the conflict point ($L^s(t)$) as a running cost. $\omega^{\cdot}$ represents the weight assigned to each component within the running cost. To facilitate conflict analysis, we introduce the concept of configuration space (c-space). In this space, each conflict point is represented by a circle, the diameter of which equals the size of the largest traffic participant involved in the interaction, enabling each traffic participant can be treated as a mass point.

$$L^s(t) = -\frac{1}{2\omega^s}\left(\frac{s_i(t)}{v_i(t)} - \frac{s_j(t)}{v_j(t)}\right)^2 \tag{5}$$

where i and j denote any two different traffic participants. $s_i(t)$ and $s_j(t)$ signify the Euclidean distance from the traffic participant i and j to the conflict point, respectively. The efficiency is evaluated by the difference between the current speed and the expected speed (v^e) ($L^v(t)$). We also define the acceleration loss ($L^a(t)$), yaw angle loss ($L^\delta(t)$) and position loss ($L^d(t)$).

$$L^v(t) = \frac{1}{2\omega^v}(v(t) - v^e)^2 \tag{6}$$

$$L^a(t) = \frac{1}{2\omega^a} a(t)^2 \tag{7}$$

$$L^\delta(t) = \frac{1}{2\omega^\delta} \delta(t)^2 \tag{8}$$

$$L^d(t) = \frac{1}{2\omega^d}(l^x(t) - l^{x_{end}})^2 + \frac{1}{2\omega^d}(l^y(t) - l^{y_{end}})^2 \tag{9}$$

where $(l^x(t), l^y(t))$ is the position of the traffic participant, and (x_{end}, y_{end}) represents the pre-specified end point for that participant. The term $\Phi(t_0 + T, x(t_0 + T))$ is defined by the position loss, indicating the deviation from the target end point at time $t_0 + T$.

In this paper, $L^s(t)$, $L^v(t)$, $L^a(t)$, $L^\delta(t)$ and $L^d(t)$ are employed for both CAVs and bicycles, and their state and control vectors are denoted as $g(t) = [l^x(t)\ l^y(t)\ \psi(t)\ v(t)]^T$ and $u(t) = [a(t)\ \delta(t)]^T$, respectively. As for pedestrians, the running cost comprises $L^s(t)$, $L^v(t)$, $L^a(t)$ and $L^d(t)$, with their state and control vectors represented by $g(t) = [l(t)\ v(t)]^T$ and $u(t) = [a(t)]^T$.

3 Results

The Frenet coordinate system is utilized to facilitate the planning of curved trajectories (Werling *et al.*, 2010). To generate the optimal trajectory for each participant within this dynamic interaction environment, Pontryagin's Minimum Principle (PMP) is employed (Hang *et al.*, 2022). An iterative method is introduced to solve it, with further detail available to interested readers in Hoogendoorn *et al.* (2023). We conducted simulation experiments in four (4) different interaction scenarios to verify the performance of the proposed DI-CDM, as shown in Figure 1. It depicts the trajectories of various traffic participants, utilizing circles of distinct colors to represent varying speeds, with numbers within these circles denoting specific time point. The DI-CDM demonstrates effective performance in proposing safe and efficient operational strategies within the boundaries of motion constraints and traffic regulations. For example, in the Figure 1(a), the interaction with the pedestrian prompts the CAV to decelerate in order to avoid the pedestrian before reaching the conflict point. After the 3rd second, the CAV continues to decelerate due to the motion constraints imposed by the turn.

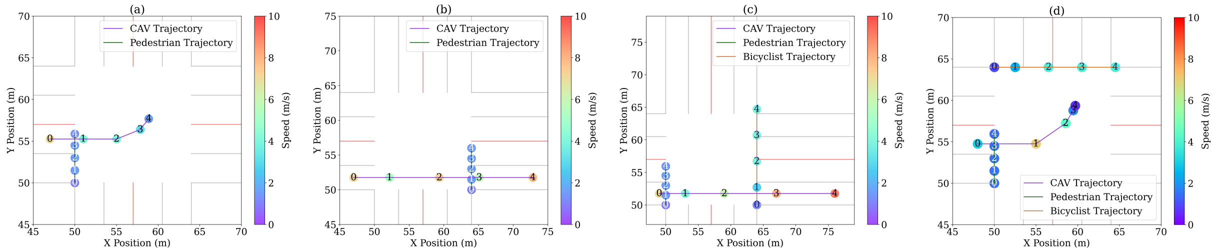

Figure 1 – *Results of DI-CDM in different interaction scenarios*

4 Discussions

Autonomous driving technology, while presenting new opportunities for the development of intelligent transportation systems, indeed raises valid concerns about the safety risks of VRUs, especially at unsignalized intersections. However, different traffic participants exhibit distinct motion characteristics, and the presence of multi-player, dynamic, and continuous interactions significantly complicates analysis. Consequently, developing a safe, harmonious, and efficient automated unsignalized intersection remain a critical challenge. This paper employs a differential game to develop a dynamic interaction-based cooperative decision-making approach for heterogeneous traffic participants to identify an equilibrium strategy for VRUs and CAVs, facilitating their conflict-free and efficient navigation through intersections. Future work will concentrate on addressing the dynamically changing locations of conflict points, due to the uncertain positions of traffic participants, which introduces additional complexity into conflict analysis.

References

Bautista-Montesano, Rolando, Galluzzi, Renato, Ruan, Kangrui, Fu, Yongjie, & Di, Xuan. 2022. Autonomous navigation at unsignalized intersections: A coupled reinforcement learning and model predictive control approach. *Transportation research part C: emerging technologies*, **139**, 103662.

Hang, Peng, Huang, Chao, Hu, Zhongxu, & Lv, Chen. 2022. Driving conflict resolution of autonomous vehicles at unsignalized intersections: A differential game approach. *IEEE/ASME Transactions on Mechatronics*, **27**(6), 5136–5146.

Hoogendoorn, Serge, & HL Bovy, Piet. 2003. Simulation of pedestrian flows by optimal control and differential games. *Optimal control applications and methods*, **24**(3), 153–172.

Hoogendoorn, Serge, Gavriilidou, Alexandra, Daamen, Winnie, & Duives, Dorine. 2021. Game theoretical framework for bicycle operations: A multi-strategy framework. *Transportation Research Part C: Emerging Technologies*, **128**, 103175.

Hoogendoorn, Serge, Knoop, Victor, Mahmassani, Hani, & Hoogendoorn-Lanser, Sascha. 2023. Game-theoretical approach to decentralized multi-drone conflict resolution and emergent traffic flow operations. *arXiv preprint arXiv:2308.01069.*

Qin, Ziye, Ji, Ang, Sun, Zhanbo, Wu, Guoyuan, Hao, Peng, & Liao, Xishun. 2024. Game Theoretic Application to Intersection Management: A Literature Review. *IEEE Transactions on Intelligent Vehicles*, 1–19.

Werling, Moritz, Ziegler, Julius, Kammel, Sören, & Thrun, Sebastian. 2010. Optimal trajectory generation for dynamic street scenarios in a frenet frame. *Pages 987–993 of: 2010 IEEE international conference on robotics and automation*. IEEE.