alpha-fair tradable credit schemes

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1 INTRODUCTION

In the past two decades, there has been a substantial increase in research concerning tradable credit schemes (TCS) as a means of managing demand to reduce congestion. However, majority of the suggested approaches rely on models where the initial distribution of credits is uniform across origin-destinations and consequently overlook the impeding consideration of the nature of allocation of initial credits. This paper presents a novel approach to TCS by introducing an alpha-fair initial distribution of credits and link specific credit charges. In this innovative framework, distributional fairness is not just a conceptual aspiration but a mathematically rigorous concept grounded in the principles laid out by renowned theorists like Nash, Rawls, and utilitarian ethics. By examining the results across these distinct fairness criteria, the paper offers a comprehensive understanding of how the allocation of initial credits impacts the scheme's efficiency and equity. This holistic perspective not only enhances our theoretical understanding but also has practical implications for the design and implementation of TCS in real-world transportation and resource allocation contexts. The alpha fair tradable credit schemes also ensures that the available capacity within the transportation network is shared fairly among the competing origin destination pairs following fair resource allocation models adopted from wireless networks.

This paper then offers a novel strategy adopted from wireless networks research to equitably distribute credits in a TCS which in turn sets fair link specific credit charges. Alpha-fairness is an overarching concept that unifies four axiomatically accepted distributional fairness criteria, and generalizes Rawlsian fairness, utilitarian ethics, and Nash's fairness in two-player games to many player games. This framework accommodates diverse perspectives on fairness by incorporating alpha as a parameter, allowing for a continuum of fairness levels. At one end, when alpha is close to infinity, it emphasizes Rawlsian fairness, ensuring that the worst-off participants receive a substantial share of the resources. On the other hand, when alpha tends toward zero, it converges towards utilitarian ethics, prioritizing the maximization of overall welfare. Additionally, when alpha is equal to one, alpha fairness encompasses Nash's fairness, which focuses on equal marginal utilities among participants in a two-player setting, ensuring that no participant can unilaterally benefit from reallocating resources. By integrating these axiomatic fairness criteria into one framework, alpha fairness provides a powerful and flexible tool for addressing a wide range of allocation and resource-sharing problems while accommodating diverse notions of fairness and equity. From the methodological aspect, the α-fairness framework is solved using Lagrange method, which guarantees an exact solution for the upper level and faster convergence of the bi-level model.

The alpha fairness approach allows the transportation planner to manipulate alpha values to tailor the specific needs of ODs or travelers/trips type and fairness objectives. This is especially important because, depending on the prevailing congestion levels in the network, a fully fair control scheme can lose its efficiency and result in low trip completion rates for parts of traffic network and vice versa. The transportation planner can then dynamically adapt the model depending on their knowledge of the congestion levels, network structure and conditions. Proportional fairness and minimum delay, especially, allow for fair solutions without much sacrificing efficiency. The suggested model therefore helps researchers and policymakers evaluate the trade-offs between different fairness criteria, optimize allocation processes, and make informed decisions about TCS implementation.

2 Methodology

The alpha-fair tradable credit scheme (α-FTCS) model is formulated as a bilevel mathematical programming problem with equilibrium constraints. The upper level solves for the initial credit distribution and link-specific credit charges. Given the TCS by the upper level, travelers' behavioral responses are modelled in the lower level to solve for the route flows and market credit price equilibrium. The two levels iterate until a solution is reached.

2.1 Alpha-fair initial credit distribution and link-specific credit charges

The upper-level allots the available capacity in the transportation network fairly between users from different ODs. The link-specific credit charges are determined by the Lagrange multipliers of the link capacity constraint. This is in line with Aalami & Kattan (2022) in which shadow price for links in a proportionally fair market is determined by the flow on the links that the ODs sent. By setting this shadow price, the links achieve a network wide fairness objective. For the α-FTCS model the links set link-specific credit charges in the upper-level objective, and the price of the credits is determined by the market equilibrium in the lower-level described in section 2.2. The upper level is then formulated as follows:

$$max \sum_{w \in W} \sum_{g \in G} a^{w,g} \frac{U_{w,g}(q^{w,g})^{1-\alpha}}{1-\alpha} \tag{1}$$

$$s.t. \sum_{w \in W} \sum_{g \in G} R_l^{w,g} q^{w,g} \leq c_l, \tag{2}$$

$$q^{w,g} \geq 0 \tag{3}$$

Where $U_{w,g}(q^{w,g})$ is the utility of OD w and user group g and $a_{w,g}$ is the weight or priority of access allocated to OD w and user group g, $q^{w,g}$is the number of credits distributed to users from OD w and traveler group g, $R_l^{w,g}$ is the assignment matrix obtained from the lower-level route flow and credit market equilibrium problem and c_l is the capacity of link l. Depending on the value of α, different notions of fairness are derived to balance between efficiency and fairness. As the value of α increases, fairness of the solution improves while efficiency deteriorates. For instance: 1) for $\alpha = 0$, we obtain max-throughput, i.e., maximum efficiency, 2) $\alpha = 1$ is equivalent to proportional fairness, 3) $\alpha = 2$ is equivalent to minimum potential delay, and 4) as $\alpha \to \infty$, max-min fairness is achieved. α-fairness concept was first introduced in transportation research by Moshahedi & Kattan, (2022) for equitable perimeter control based on the macroscopic fundamental diagram. The equivalence of this formulation to a fair network resource allocation can be found in (Aalami & Kattan, 2017).

The LaGrangian for (1), (2) and (3) can be written as:

$$L(q:p) = \sum_{w\in W}\sum_{g\in G} a^{w,g}\frac{U_{w,g}(q^{w,g})^{1-\alpha}}{1-\alpha} - \sum_{l\in L}\pi_l^g\left(\sum_{w\in W}\sum_{g\in G} R_l^{w,g}q^{w,g} - c_l\right) \quad (4)$$

where π_l are the LaGrange multipliers equal to the link-specific credit charges.

2.2 Route flows and credit market price equilibrium

Assuming a logit route choice model and a linear income utility function:

$$u_k^{w,g} = \alpha^g.T_k^w - p.\left(q^{w,g} - \sum_{l\in L}\delta_{k,l}\pi_l^g\right) + \varepsilon_k^{w,g} \quad (5)$$

$$T_k^w = \sum_{l\in L}\delta_{k,l}t_l \qquad \forall k \in K^w,\; w \in W \quad (6)$$

where $v_k^{w,g}$ is the deterministic observable portion of the utility for path k between OD w and for user group g, T_k^w is the travel time on path k for OD w, p is the unit credit price in the credit market, $\delta_{k,l}$ is the link-path incidence value (0 or 1), $\varepsilon_k^{w,g}$ is a random error term and t_l is the travel time on link l.

Theorem: The flow distribution $(f) \in \Phi$ and market price $(p) \in R^+$ are in network and market equilibrium under a given feasible tradable scheme if they solve the following Variational Inequality (VI) problem:

$$\sum_{g\in G}\sum_{w\in W}\sum_{k\in K^w}\left(v_k^{w,g} + \theta^{w,g}lnf_k^{w,g*}\right).\left(f_k^{w,g} - f_k^{w,g*}\right) +$$

$$\left(Q - \sum_{g\in G}\sum_{w\in W}\sum_{k\in K^w}\sum_{l\in L}\delta_{k,l}\pi_l^g f_k^{w,g*}\right).(p - p^*) \geq 0 \quad \forall (f,p) \in \Phi \quad (7)$$

where the feasible region Φ is defined by:

$$\sum_{k\in K^w} f_k^{w,g} = d^{w,g}, \quad \forall\, w \in W, g \in G \quad (8)$$

$$p \geq 0 \quad (9)$$

$$f_k^{w,g} \geq 0 \quad (10)$$

Proof of the theorem is available in the full paper.

3 Preliminary Results

A small network with 2 ODs (AB) and (AC) shown in Figure 1 is considered to preliminarily examine the performance of the α-FTCS model. The weights a_i are 2 and 1, and the demands are 200 and 300 for ODs AB and AC respectively.

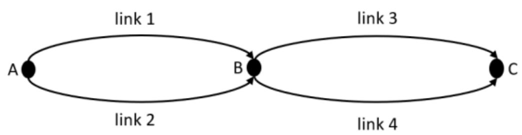

Figure 1 – *Illustration of Sample Network*

The link properties which are inputs to the model are shown in Table 1.

Table 1 – *Link properties*

Link(l)	Capacity(c_l)	Free flow time($\frac{l_l}{s_l}$)
1	110	10
2	100	12
3	90	7
4	100	5

The results of the TCS model are shown in Table 3. The bi-level model converges in less than 7 iterations for all values of α.

Table 2 – *Preliminary results for the sample network*

Name	α -value	Objective function	Physical Meaning	Objective Value	Initial distribution	Link-specific credits
Max-throughput	$\alpha = 0$	$\sum_{w \in W} a^w q^w$	Maximize total number of demands served	220	$q^{AB} = 110$ $q^{AC} = 100$	$\pi^1 = 2$ $\pi^2 = 0$ $\pi^3 = 0$ $\pi^4 = 0$
Proportional fairness	$\alpha = 1$	$\sum_{w \in W} a^w \log(q^w)$	Make the flow of passengers for each OD and their trip cost inversely proportional	14.006	$q^{AB} = 110$ $q^{AC} = 100$	$\pi^1 = 0.0182$ $\pi^2 = 0.005$ $\pi^3 = 0$ $\pi^4 = 0.005$
Minimum-delay	$\alpha = 2$	$\sum_{w \in W} \frac{a^w q^{w-1}}{-1}$	Minimize potential delay of flows	-0.0282	$q^{AB} = 110$ $q^{AC} = 100$	$\pi^1 = 0.1653$ $\pi^2 = 0.05$ $\pi^3 = 0$ $\pi^4 = 0.05$
Max-min fairness	$\alpha \to \infty$	$\sum_{w \in W} \frac{a^w q^{w-\infty}}{-\infty}$	Divide resources equally	0	$q^{AB} = 100$ $q^{AC} = 100$	$\pi^1 = 0$ $\pi^2 = 0$ $\pi^3 = 0$ $\pi^4 = 0$

4 Discussion

Fairness within transportation systems is often assessed by examining how travel demand management schemes distribute both benefits and losses across Origin-Destination pairs (ODs) and traveler groups. The preliminary evaluation for this model considers the uniformity of the initial distribution of credits and link costs. A higher initial credit distribution enhances the utility of an OD, while increased link costs correspond to greater disutility for flows passing through those links. The findings presented in Table 2 highlight that schemes aimed at achieving proportional fairness and minimum delay result in a more uniform distribution of link-specific credit charges across the entire network. This observation reinforces the hypothesis initially posited in this study and underscores the practicality of the bi-level model in converging towards optimal solutions within a reasonable number of iterations and with computationally efficient processing times.

To achieve a more comprehensive understanding of fairness within this context, we will further explore in the full paper more analyses of the results using concepts such as utility uniformity and envy-freeness proposed by Feldman et al. (2009). We will extend the analyses to a larger network to more comprehensively assess the distribution of benefits and burdens to different ODs and traveler groups within the transportation network facilitated by the designed TCS.

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