Assessing the Conditional Value-at-Risk of a train schedule under fuzzy activity duration

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1 INTRODUCTION

As any complex system, rail networks are vulnerable to delays and disruptions, caused by operational challenges, delay propagations, adverse weather events and technical failures. In real-time, rescheduling actions are taken to maintain operational integrity and minimize disruptions to service. Due to strict computational time constraint rescheduling is usually carried out neglecting the presence of uncertainties with a substantial underestimation of the risks. Hence, there is the need to correctly evaluate the risks to allow the dispatcher to take informed decisions and trust the solutions approaches.

In real contexts, it is not always possible to assume that activity duration times are deterministic and known in advance, as uncertainties may have a significant impact. The uncertainty in activity durations can be represented as random variables, fuzzy numbers, or intervals, and in the literature there is no universally accepted risk measure (Kou & Peng, 2014, Rockafellar & Royset, 2013). Although train rescheduling is a relevant and well studied problem, in the literature there are not many studies focusing on the evaluation of uncertainties associated to a schedule (Zhan *et al.*, 2024) and even fewer associated to the assessment of the risk of a train schedule (Meloni *et al.*, 2021). This is probably due to the computational difficulties connected to scenarios and sampling methods.

Given a solution for the real-time train rescheduling problem computed using deterministic methods, we asses the risk of its quality worsening in presence of uncertainty on the dwell time duration. We assume that the dwell times are integers, their uncertain values are represented by fuzzy numbers and the realised duration of an activity is only known after its completion. We explore the evaluation of Conditional Value-at-Risk of the makespan in a temporal network with fuzzy activity durations on the arcs.

2 ACTIVITY NETS WITH FUZZY TEMPORAL VALUES

It is well known that a schedule can be represented by temporal activity networks and a commonly used performance measure in activity scheduling problems is the Makespan $\mathbf{C}_{max}$ or equivalently Maximum Lateness $\mathbf{L}_{max}$. Let us model a solution for the train rescheduling problem using a *fuzzy temporal activity networks* (*FTANs*) (Dellino *et al.*, 2022), temporal activity nets with fuzzy valued uncertain durations. Uncertain quantities will be represented in bold. A *FTANs* is a pair $(G, \mathbf{D})$, where $G = (N, A)$ is a directed acyclic graph representing the activity precedences, the set N of nodes is associated with events, the set A of m arcs representing the activities, and $\mathbf{D} = (\mathbf{D}_1, \ldots, \mathbf{D}_m)$ is the vector of fuzzy durations associated with the activities.

All quantities depending on the activity execution times are fuzzy quantities, including starting and ending time of each activity and makespan $\mathbf{C}_{max}$ (Dubois *et al.*, 2003, Hapke & Slowinski, 1996). A fuzzy quantity is a fuzzy set of the real line $\mathbb{R}$. A fuzzy set M of the universe of values X is characterised by a membership function μ_M taking value in the $[0, 1]$ interval. For each element $x \in X$, $\mu_M(x)$ defines the degree to which x belongs to $M = \{x \in X; \mu_M(x) \in [0, 1]\}$. An α-cut of M is the crisp set $M_\alpha = \{x \in X | \mu_M(x) \geqslant \alpha\}$.

The duration of all the activities is a fuzzy number which is defined as a bounded support fuzzy quantity whose α-cuts are closed intervals. We represent membership function of the activity durations based on the full breakpoints ordered sequence (Fortemps, 1997). The decomposition by α-cuts (Nguyen, 1978) can be used as a general approach to compute functions of activity durations as $\mathbf{C}_{max}$. According to this method, the membership function $\mu_F(x)$ of $F(\mathbf{D})$ can be reconstructed from its α-cuts F_α as $\mu_F(x) = max\{\alpha : x \in F_\alpha\}$.

3 COMPUTING RISK INDICES FOR THE MAKESPAN

As risk index we consider the Conditional-Value-at-Risk of the makespan at probability level γ ($\mathrm{CVaR}_\gamma(\mathbf{C}_{max})$), also known as Expected Shortfall. $\mathrm{CVaR}_\gamma(\mathbf{C}_{max})$) can be defined as the average over the worst γ% cases. Meloni & Pranzo (2020) and Meloni & Pranzo (2023) introduced a counting methodology to compute the expected shortfall of the makespan in scheduling problems represented by activity networks where the processing times are integers and their equally possible values belong to known intervals. This approach has been extended to *FTANs* in Dellino *et al.* (2022).

The algorithmic scheme for the case of crisp interval durations introduced in Meloni & Pranzo (2020) and Meloni & Pranzo (2023) can be summarized as follows:

- The conceptual scheme of the algorithms is a counting approach working backward. Starting from the pessimistic makespan level, it counts the number of configurations leading to makespan L. Then L is decreased and a new counting occurs. The iterations stop when enough information is gathered to compute the risk indices at the desired probability level γ.

- The counting step at a level L is done by considering the possible reduction transformations on a specific critical subgraph (Meloni & Pranzo, 2020, 2023). If all the considered critical subgraphs completely reduce by series and parallel transformations, then the proposed approach computes the exact risk indices at level γ, otherwise an interval ($[LB_\gamma, UB_\gamma]$) of the considered indices is returned.

In Dellino *et al.* (2022) the algorithmic scheme for crisp instances is extended to consider fuzzy durations by decomposing the membership functions of the activity durations into a finite number of α-cuts. For each selected level α, each fuzzy duration is cut at level α. This phase produces a set of crisp instances (activity networks with interval valued durations). An estimate of the indices of interest is determined as the average of the related LB_γ and UB_γ obtained by the algorithm at each α-level. This method is simple to implement but it could be intractable if ran for too many α-cuts.

In this work we introduce three different *Decomposition Management Methods* (DMM) to control the number of evaluated α-cuts:

- The first method, *Uniform Sampling* (US(δ)), uses all the α values uniformly taken in the interval $[0, 1]$ with a resolution of δ as step value. This method produces the best possible results at the cost of a high running time;
- The second DMM, named *Breakpoint* (BP) uses the whole set of α values corresponding to the breakpoints of all the fuzzy activity durations received as input;
- The third proposed DMM is indicated as *Budget Constrained* (BC(δ, B)). To obtain the α values, BC adopts a progressive sampling of the set of values in the interval $[0, 1]$ at step δ. The method has a computational budget available in terms of the maximum number B of α-cuts to consider. At each iteration, a new single additional α value is identified on the basis of an estimate of the best achievable increment for the quality of the decomposition. BC stops either when the budget ends or because the decomposition cannot be further improved by adding more α-cuts.

To compare two fuzzy values associated with a risk index we adopt a method based on the representation of fuzzy numbers as α-level sets (Sevastjanov & Róg, 2006). It returns the probabilities $P(B > A)$, $P(B = A)$ and $P(B < A)$ for each pair of fuzzy numbers A and B.

4 RESULTS

On the basis of the computational results reported in Meloni & Pranzo (2023), we adopt the algorithmic variant named Sl/Adv/mS which is able to determine a fast and extremely good estimation of CVaR of the makespan. We use as test case 189 distinct rescheduling solutions obtained for 24 different perturbed instances of the 2008 timetable of the railway network around the central station of Utrecht (NL), which considers Utrecht, the busiest station in the Netherlands, and its 5 main traffic directions, branching toward Amersfoort, Arnhem and Germany, Den Bosch, Amsterdam, and finally Rotterdam and The Hague. Each instance considers a time window of one peak hour for a total of 79 running trains.

Table 1 – *Results*

DMM	γ	$P(\text{US}(0.01) > *)$	$P(\text{US}(0.01) = *)$	$P(\text{US}(0.01) < *)$	T (s)	#α-cuts
US(0.01)	0.01	0	1	0	6.44	101
	0.05	0	1	0	6.52	101
	0.1	0	1	0	6.54	101
BP	0.01	0	0.68	0.32	0.19	3.0
	0.05	0	0.68	0.32	0.19	3.0
	0.1	0	0.71	0.29	0.19	3.0
BC(0.01,7)	0.01	0	0.89	0.11	0.23	3.7
	0.05	0	0.91	0.09	0.23	3.7
	0.1	0	0.90	0.10	0.24	3.7
BC(0.01,18)	0.01	0	0.96	0.04	0.47	7.4
	0.05	0	0.96	0.04	0.48	7.4
	0.1	0	0.96	0.04	0.49	7.4
BC(0.01,∞)	0.01	0	1	0	0.68	10.6
	0.05	0	1	0	0.70	10.6
	0.1	0	1	0	0.70	10.6

In Table 1 we summarize the results of the computational campaign where we compare 5 DMMs. Specifically, we set US(0.01) as the reference DMM and we compare BP and 3 versions of BC against it. In the first two columns we report the adopted manager and the γ value, respectively. In the next 3 columns we compare US(0.01) with the other DMMs, in terms of the probability of US(0.01) being better, equal or worse than the DMM the row refers to, here called $\circledast$. Finally the last 2 columns report, on average for each train rescheduling solution, the total computation time and the average number of required α-cuts.

As expected, column P(US(0.01)>B) is always 0 since by construction US(0.01) cannot be improved by other managers. On the other hand, its computation times and number of analyzed α-cuts are much higher than for the other DMMs. Whereas the increase of γ reflects only on a marginal increase of CPU time as uncertainty grows and crisp instances are harder to solve.

BC(0.01,∞) is able to perfectly match US(0.01) quality at a fraction of the required time and α-cuts. The remaining 3 DMMs require even shorter CPU times at the expense of a faulty reconstruction ($P(\text{US}(0.01) < \circledast) > 0$), but overall, none of the proposed approaches is dominated.

The results show how the proposed DMMs are able to estimate the fuzzy CVaR within short CPU times and with excellent quality (all the crisp instances are in fact solved to optimality). As future research we highlight the possibility to extend this approach to different quantile-based risk measures (as VaR) and the development of more advanced DMMs schemes.

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