Augmented ε-Constraint-Based Optimization for Multi-Objective Multi-Modal Transport Networks Management

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1 INTRODUCTION

The increasing urbanization, combined with shrinking space for transport infrastructure and private parking, significantly challenges urban accessibility. Moreover, the rising number of vehicles exacerbates congestion in city centers, leading to longer commute times, increased noise levels, and greater air pollution. These issues underscore the urgent need for creating low-car urban zones. One promising approach is an integrated traffic management system that considers various modes of transportation—such as cycling, walking, shared mobility, and public transport. However, multi-modal traffic management typically involves diverse stakeholders with potentially conflicting interests, which necessitates a balance of these interests through multi-objective optimization. Traditional approaches often employ a weighted sum method to transform multiple objectives into a single objective. This method significantly constrains the solution space and complicates the assignment of appropriate weights to different objectives. Therefore, generating a Pareto front for multi-modal traffic management could provide decision-makers with a set of efficient solutions, enabling them to select the most suitable option. The ε-constraint method is recognized for its ability to generate a Pareto front. The question we discuss here is whether this method can be effectively applied to managing multi-objective, multi-modal traffic networks.

In this study, we answer this question by proposing an augmented ε-constraint-based optimization framework for multi-objective multi-modal traffic management. This framework is bi-level and can accommodate various traffic models and objectives that reflect the diverse interests of multiple stakeholders. Thus the multi-modal traffic management problem can be formulated as a multi-objective nonlinear optimization problem. The augmented ε-constraint method (Mavrotas, 2009) is employed to efficiently address the multiple objectives, and the multi-start sequential quadratic programming method is used to solve the nonlinear optimization problems, such that the Pareto front is obtained. We validate the effectiveness of our framework through a case study, whose preliminary results show that our method improves the traffic performance and provides insights into the trade-off among different objectives.

2 Methodology

To effectively coordinate various traffic modes (such as private cars, buses, and metro) and integrate multiple traffic management measures (including speed limits, traffic signals, and road pricing), a comprehensive top-down framework is essential. This framework will allow us to study the impact of different traffic control measures on user behavior and overall traffic system performance, which makes it possible to analyze and determine the best policies for decision-makers, by optimizing multiple objectives like minimizing total travel time, reducing traffic operation costs, and decreasing the number of private cars.

2.1 Bi-level optimization framework for multi-modal traffic management

The proposed framework is as shown in Figure 1. In the low-level user-equilibrium (UE) optimization, travel cost on each path or link is formulated as a function of the traffic flow on the

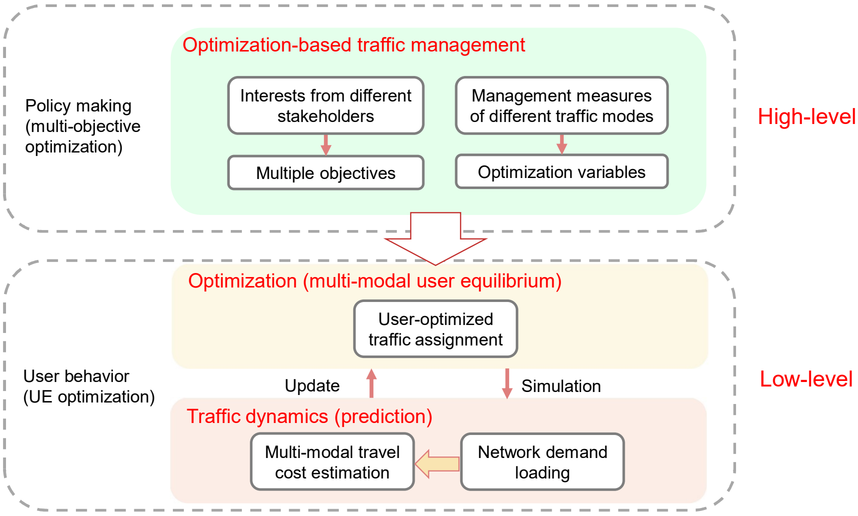

Figure 1 – *The bi-level optimization framework for multi-modal transport management*

network. Then, with the given travel demands, the travel cost on each path can be estimated, and the user-optimized traffic assignment is performed to achieve a UE, which represents the user behaviour when choosing routes, under rationality assumptions. In a UE, the users choose their routes selfishly to minimize their own travel cost, which leads to the situation where the journey times in all routes actually used are equal and less than those that would be experienced by a single vehicle on any unused route (Wardrop, 1952). Note that there can be a gap between the UE and the system optimum (in which the total travel cost is minimized). This implies that the network-wide traffic performance can be further improved by guiding the users through some traffic control measures (e.g., road pricing, see Roughgarden & Tardos (2002) for more details), and these control measures can be formulated as optimization variables and optimized in the high-level module of the framework. In addition to the system optimum, various objectives can be considered simultaneously in the high-level module.

Different to existing work that utilizes similar structure (see e.g. Du *et al.* (2022)), we believe ours is the first attempt to employ such a bi-level framework for **multi-objective multi-modal** transport network management. There is already research available on the travel cost estimation in a multi-modal transport scenario (Pi *et al.*, 2019, Du *et al.*, 2022). According to the specific formulation of the travel cost function, the solution method to the UE is different. If the cost function is separable, the UE problem can be formulated as an optimization problem (Beckmann *et al.*, 1957). Otherwise, the UE problem can be formulated as a variational inequality problem (Smith, 1979). For both cases, there are existing efficient algorithms for the low-level module. Therefore, the multi-objective multi-modal traffic management problem can be formulated as a multi-objective optimization problem, which can be solved by the method presented in the following section.

2.2 Augmented ε-constraint-based multi-objective optimization

Due to the low-level optimization problem for the UE calculation, the high-level problem is highly nonlinear and nonconvex, which makes it challenging to find the optimal solutions. In this study, we employ an advanced method: the augmented ε-constraint method (Mavrotas, 2009), to address the multiple objectives. Compared to conventional ε-constraint method, this approach avoids the production of weakly Pareto optimal solutions and accelerates the whole process by avoiding redundant iterations, enabling us to obtain the Pareto front more efficiently. Alongside the augmented ε-constraint method, we use the multi-start sequential quadratic programming (SQP) method to solve each single nonlinear optimization problem and to approximate the global optimum. The details of the solution algorithm are omitted here due to limited space.

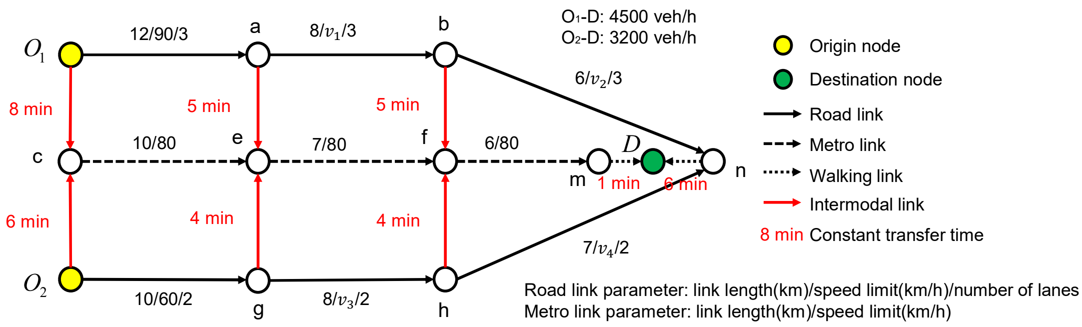

Figure 2 – *Multi-modal transport network and the parameters used in the case study*

The objectives in the high level can be designed according to the interests of different stakeholders. In this study, the following objectives are considered: 1) The total time spent (TTS) of all the users, which represents the public interest that aims at improving the system-wide performance; 2) The total private car flow, which represents the interest of the citizens who wish to reduce the number of private cars entering the city center due to limited parking space, or noise and air pollution; this values needs to be minimized; 3) The operation cost of public transit, which represents the interest of the public transport operator; this objective needs to be minimized to ensure economic viability; a more rigorous formulation would have the public transport operator as an independent participant in a more complex game theoretical scenario. Here for the sake of initial exploration we consider a single network management/design decision maker. Note that the proposed solution method is general and can deal with different objectives.

3 Results and discussions

A case study is performed on an example network (see Figure 2) to illustrate multi-objective multi-modal traffic management. The network consists of three modes: road link for private cars, metro link, and walking link. There are two origin nodes O_1, O_2 and one destination node D, resulting in two OD pairs. The users departing from O_1 and O_2 can choose to transfer to metro at the beginning or in the middle nodes a, b, g, h, or they can finish the entire trip without changing traffic modes. For simplicity, the travel cost of both mode links are formulated as strictly increasing functions dependent on the link flow, and we assume the cost function to be separable. The reader can refer to Beckmann *et al.* (1957) for more details about the problem formulation. The optimization variables in the high-level module include the speed limit on the roads link (v_1, v_2, v_3, v_4) and the metro departure frequency. The operation cost is also a increasing function dependent on the metro frequency.

By performing the bi-level optimization algorithm on the multi-modal network with the three specified objectives, we can obtain the Pareto front, as shown in Figure 3, where the points denote the obtained solutions with a coarse objective grid. The Pareto front presents a trade-off among these three objectives, which implies that the improvement in one objective is always at the cost of deteriorating the other objectives. Compared with the case without high-level traffic management, the TTS can be reduced by up to 30.443%, while the total private car flow can be reduced by as much as 72.84%. Note that there is a 2D Pareto front between any two of these objectives. The framework can also be used to evaluate the reliability of a network. For example, we can study how the management method can maintain the traffic performance and minimize the influence when a link is disrupted. This experiment is omitted here due to limited space.

4 Conclusions

In this work, we propose an augmented ε-constraint-based bi-level optimization framework for multi-objective multi-modal traffic management, in order to address the most urgent traffic problems that cities are up against, such as creating low-car areas, and meanwhile address

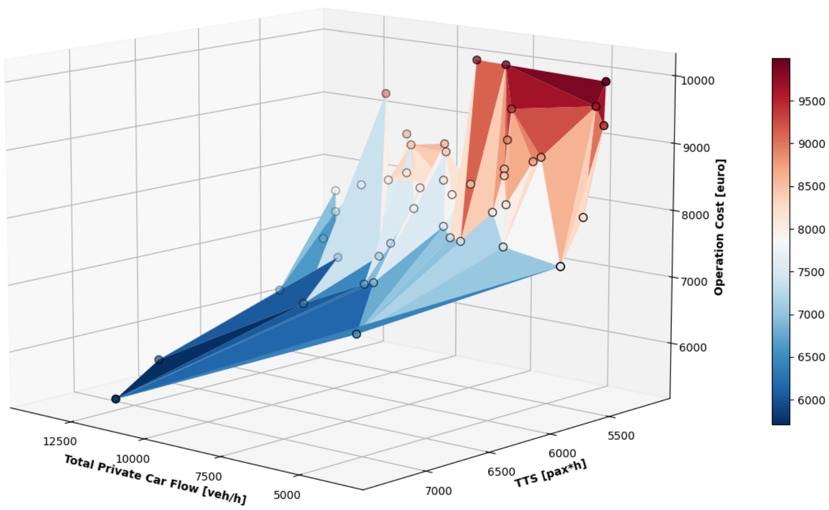

Figure 3 – *The 3D Pareto fronts of TTS, total private car flow, and operation cost objectives in the case study*

other concerns from different stakeholders. Preliminary results validate the effectiveness of the proposed method. The proposed framework can be used to address a wide range of multi-modal transport problems with different scenarios and objectives. In the future, advanced solution algorithms can be developed based on the method proposed in this work, to solve the optimization programming for specific problems. In addition, an interactive process can be introduced to enable the decision maker to find his/her preferred solution, by iteratively generating more refined Pareto front around the selected solution. Furthermore, large language model (LLM) techniques can be introduced in the interactive process to interpret the technical results (e.g., Pareto front) to the decision maker, which can further facilitate the decision process.

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