Congestion-aware optimization of school start times: A macroscopic approach

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Extended abstract submitted for presentation at the Conference in Emerging Technologies in Transportation Systems (TRC-30)
September 02-03, 2024, Crete, Greece

April 29, 2024

Keywords: Traffic Congestion, Demand Management, School start time selection, Optimization.

1 INTRODUCTION

Road traffic congestion refers to a condition on road networks, often occurring during peak hours, where traffic demand exceeds the available capacity of the roads, with severe economic and environmental footprint (Papageorgiou *et al.*, 2003). A crucial aspect of the morning commute lies within the associated trips toward the underlying schools. A portion of commuters are forced to stop first at the school their children belong to before going to their work (Jia *et al.*, 2016). In principle, different categories of commuters compete for space with the *school-oriented* commuters. Due to the congestion that they anticipate to emerge in their routes, school-oriented commuters start their morning trip in a synchronous manner (He *et al.*, 2022). As an outcome, the network is heavily loaded with vehicles, eventually forming a peak demand that the network cannot directly accommodate (Daganzo, 2007).

To address the issue of peak-demand formation during the morning period, *staggered school schedules* appeared as a promising demand management mechanism. This paradigm considers that the start time of the schools is subject to change, causing a temporal redistribution of traffic flow by modifying the departure time of commuters who head to the respective school (Bertsimas *et al.*, 2019). A limitation identified in the morning commute literature is that the congestion aspect is a less examined property, raising the need for a staggered school schedule approach that can handle congestion. In the context of macroscopic traffic modeling, there are three traffic quantities of interest, namely, speed, flow, and density, which are used to build the Macroscopic Fundamental Diagram (MFD) (Daganzo & Geroliminis, 2008). The utilization of the MFD considers aggregate traffic measurements and leads to a less computationally intensive representation of traffic compared to highly complex detailed microscopic models (Newell, 1987).

Building upon our previous work, Georgantas *et al.* (2023), in this paper, we extend the modeling framework involving the movement of multiple classes of commuters in an urban traffic network architecture of arbitrary size. In this context, the central point of this paper is the construction of a demand management mechanism that staggers the start time of schools, taking into account the congestion aspect captured through the regional and class-specific traffic dynamics. The resulting formulation leads to a bi-objective mixed integer nonlinear program (MINLP) that jointly minimizes i) the Total Time Spent (TTS) of all vehicles inside the network and ii) the associated overall delay observed between the initial and the staggered start times of all schools. The complete outline of the problem corresponds to highly non-convex and nonlinear traffic dynamics, where a derivative-based optimization algorithm could prove inefficient. To deal with this issue, we develop a feedback-based scheme that blends the dynamics dictating the related traffic system with the *surrogate* (black-box) optimization solver.

2 METHODOLOGY

We examine an urban traffic network consisting of multiple homogeneous regions. In this network, we assume that two classes of vehicle commuters utilize the network infrastructure during the morning rush hour. Commuters of class H first need to drop off their children in the region where their school is located (intermediate destination) before heading to their final destination (e.g., head to work or return home). Commuters of class W head directly to their destination. When the schools start at the same time, parents (commuters of class H) have to depart from their home at a time that will allow them to arrive at the school punctually. The co-existence of both classes of commuters in the network causes a massive inflow of demand, resulting in a value of peak demand that the network cannot accommodate. To resolve this issue, our goal is to shift the start times of individual schools to redistribute the peak demand, opting to improve the traffic conditions in the network.

2.1 Regional Traffic Dynamics

In this section, we will present the mathematical framework that will account for the traffic dynamics concerning the homogeneous regions present in the traffic network.

2.1.1 Macroscopic Traffic Flow Model

Let an urban road traffic network consist of $|R|$ homogeneous regions, where $\mathcal{R} = \{1, \dots, |R|\}$. The network consists of a set of origin regions $\mathcal{O} \subseteq \mathcal{R}$ from where vehicular flows start their journey. The time horizon is quantized into time slots $k \in \mathcal{K}$ of duration T (min). Let $\mathcal{K} = \{0, 1, \dots, K\}$ denote the discrete time-slots of the entire morning commute period. We adopt the notion of the Macroscopic Fundamental Diagram (MFD), in which the *actual flow* that is exchanged between neighbouring regions, is limited by the inter-boundary capacity term (Sirmatel & Geroliminis, 2018).

2.2 Demand Management Framework

In this section, we describe the demand scheme that will regulate the start time of each school located in the network so that the overall demand that enters the network is flattened. Let $\mathcal{B} \subseteq \mathcal{R}$ denote the set that contains the regions in which schools are located. Furthermore, $\mathcal{S}_b$ denotes the set that contains the schools situated in intermediate destination $b \in \mathcal{B}$. Class H is associated with a demand $d^H_{o,s}(k)$ (veh), which expresses the number of vehicles requesting to enter the network at time-step k from origin $o \in \mathcal{O}$ toward school $s \in \mathcal{S}_b, b \in \mathcal{B}$. With τ_s (AM), we will denote the start time of school $s \in \mathcal{S}_b, b \in \mathcal{B}$ with an associated demand reading, $\mathbf{d}^H_{o,s} \in R^{K\times 1}$ (veh). Furthermore, let $x_s \in Z^+$ represent the decision variable of interest that denotes the number of time-slots that we shift the start time of school $s \in \mathcal{S}_b, b \in \mathcal{B}$. When a value x_s is selected for school $s \in \mathcal{S}_b$, then the school's start time will be shifted later in the time domain by x_s time-steps. Then the new reading for the distribution of demand with respect to commuters of class H is denoted with $\tilde{\mathbf{d}}^H_{o,s} \in R^{K\times 1}$ (veh). The relationship between vectors $\mathbf{d}^H_{o,s}$ and $\tilde{\mathbf{d}}^H_{o,s}$ is shown in Eq. (4), which shows the shifted demand values concerning commuters of class H that originate from o heading towards school s when the start time of the school is shifted by x_s time-slots

$$\tilde{d}^H_{o,s}(k) = d^H_{o,s}(k - x_s). \tag{1}$$

2.3 Problem Formulation

In this work, we are interested in minimizing two objective functions of interest: i) the Total Time Spent, (J_{TTS}) of all vehicles in the network and ii) the Start Time Change, (J_{STC}) capturing the overall change between the initial and the actual school start time. We assign weighting factors w_1, w_2 to the Total Time Spent (TTS) and Start Time Change (STC) objective functions to examine the trade-off in terms of the two objective metrics of interest. Hence, we combine the two objective functions into a single one through the scalarization technique known as the weighted sum method (Marler & Arora, 2010). In doing so, the *Combined Objective Cost* that we minimize is $J_{\text{COC}} = w_1 J_{\text{TTS}} + w_2 J_{\text{STC}}$. We have developed a paradigm that captures the traffic dynamics associated with each class of commuters separately. Due to space limitation, we do not show the analytical derivation here; however, the critical observation is that the demand management Eq. (2) is combined with the traffic dynamics associated with commuters of class H. The resulting mathematical formulation that describes the regional traffic dynamics along with the class-specific traffic dynamics corresponds to a mixed integer nonlinear program (MINLP).

2.4 Solution Approach

In this section, we provide the outline of our solution approach. First, we consider that the transport network serves as the physical plant. When a start time is selected for each school, then we execute the equations that dictate the physical plant, and we obtain as output the values of the two objective metrics mentioned above, i.e., J_{TTS} and J_{STC}. Subsequently, we can calculate the combined objective cost value, $J_{\text{COC}} = w_1 J_{\text{TTS}} + w_2 J_{\text{STC}}$ for different values of weights w_1 and w_2, respectively. Then J_{COC} is given as input to the traffic controller (optimization solver). The controller solves the related MINLP considering the underlying constraints of the physical plant and provides the control decision x_s as output. Towards this direction, we have employed a feedback loop architecture in which the control decision x_s derived from the traffic controller is fed to the physical plant. This procedure is performed in an iterative manner until J_{COC} converges. We utilize the *surrogate* optimization solver to deal with the relevant MINLP and obtain the actual start time of each school (Inc., 2024).

3 RESULTS

To evaluate the proposed framework, we consider an urban network comprising eight homogeneous regions, where the traffic dynamics of each region follow a triangular MFD model. The simulation horizon is set from [7:00 - 8:50] AM, and the simulation time step is set equal to $T = 30$ s. Furthermore, 12 schools and 25 works are scattered across different regions in the network. The initial start time of all schools is set to $\tau_s =$ 7:40 AM. In addition, we consider that $0 \leq x_s \leq 40, \forall s \in \mathcal{S}_b, \forall b \in \mathcal{B}$. Table 1 shows the obtained values for the two objective functions of interest, along with the derived school start time vector and execution time, respectively, when we solve the corresponding MINLP utilizing the surrogate solver. For the no-control case, the Total Time Spent (TTS) obtains a value equal to $J^{\text{NC}}_{\text{TTS}} = 111.10$ (veh h), (NC) standing for No-Control, with an associated value of Start Time Change, $J^{\text{NC}}_{\text{STC}} = 0$ (min), as no shifting in the start time of schools is allowed. A finding stemming from Table 1 states that the black-box solver can pinpoint particularly fast a solution that shifts the start times of only three schools out of the total twelve (causing a 40-minute total delay in the start time of schools, $J_{\text{STC}} = 40$), leading to an approximately 11% reduction in the value of Total Time Spent metric compared to the no-control case.

Table 1 – *Output of Surrogate optimization solver for different values of weights w_1 and w_2.*

Combination of Weights $w_1 = 1.0$ & $w_2 = 0.0$			
J_{TTS} (veh h)	J_{STC} (min)	Derived Pair - (AM)	Execution time (h)
99.95	40	$(\underbrace{7:40, 7:40}_{\text{Region 4}}, \underbrace{7:40, 7:53, 7:40}_{\text{Region 5}}, \underbrace{7:40, 7:40, 7:52, 7:40}_{\text{Region 7}}, \underbrace{7:55, 7:40, 7:40}_{\text{Region 8}})$	0.04

4 DISCUSSION

This paper proposes a demand management framework for multi-region urban networks with traffic dynamics dictated by MFDs. In this context, a non-convex optimization scheme is devised to determine the shifted school start times that lead to mitigation of congestion. We developed a feedback loop architecture combining traffic system dynamics with a black-box (derivative-free) optimization mechanism. The simulation experiments verify that our approach can mitigate the congestion while causing a shift in the start time of schools that does not significantly disrupt the Status Quo of the system. A potential extension of this work could be the construction of a relaxation paradigm in which each nonlinear and non-convex constraint associated with the traffic system will be approximated by a block of linear constraints, yielding a Mixed Integer Linear Program (MILP) formulation.

5 ACKNOWLEDGEMENTS

The work of Antonios Georgantas, Stelios Timotheou and Christos G. Panayiotou is supported in part by the European Union's Horizon 2020 research and innovation programme under Grant 739551 (KIOS CoE), in part by the Government of the Republic of Cyprus through the Directorate General for European Programmes, Coordination, and Development.
Antonios Georgantas is partially supported by the Foundation for Education and European Culture, Founders Nicos & Lydia Tricha, Greece.

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