Mitigating traffic congestion via ride-sharing: An optimal ride-matching scheme

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1 INTRODUCTION

A ride-sharing system allows drivers and riders with overlapping trips to travel together and share costs. In addition to economic incentives for both participants, ride-sharing holds great potential to alleviate traffic congestion and reduce emissions. At the core of a ride-sharing system is the matching problem. Different from previous studies that propose matching mechanisms to maximize total trip utilities (Tafreshian *et al.*, 2020), this study takes the perspective of the traffic manager who aims to minimize system-wise traffic congestion. Hence, it is also different from existing traffic equilibrium models that generalize ride-sharing behaviors (Di *et al.*, 2017, Li *et al.*, 2020). The proposed matching scheme is largely inspired by several recent studies (Zhang & Nie, 2018, Chen *et al.*, 2020, Farahani *et al.*, 2021), which show that the traffic network can deviate from user equilibrium (UE) and closely approach system optimum (SO) by rerouting a small fraction of travelers. Instead of assuming fully controllable autonomous vehicles (Zhang & Nie, 2018, Chen *et al.*, 2020) or arbitrarily introducing intermediate checkpoints to trips (Farahani *et al.*, 2021), we consider travelers detoured from their UE paths as ride-sharing drivers. Accordingly, the detours hold particular meanings of picking up riders and trips after dropping off riders.

In this study, we consider three groups of travelers: ride-sharing riders, ride-sharing drivers, and solo drivers. The problem is formulated as a Stackelberg game (Stackelberg, 1952), or mathematically, a mathematical program with equilibrium constraints (MPEC) (Dempe, 2003). Specifically, the traffic manager is considered *the leader*, who performs ride-matching to minimize total traffic congestion while producing reasonable matching outcomes. The leader's decision essentially manipulates the demand pattern of travelers, or *the follower*, whose routing behaviors are described by a static traffic equilibrium. The key advantage of this formulation over existing ride-sharing traffic equilibrium models (Di *et al.*, 2017, Li *et al.*, 2020) is that it decomposes the matching and routing problems through the bi-level framework and thus largely reduces the modeling complexity. Furthermore, we show that the upper-level ride-matching problem can be reformulated as an assignment problem over a hyper-network with link costs specified using the equilibrium sensitivities derived from the lower-level given current solution. Therefore, the subproblems at both levels can be solved using classic algorithms for static traffic assignment.

This study contributes to the literature from several aspects. First, we propose a novel traffic management strategy via ride-sharing and explore its potential to push the system from UE toward SO. Secondly, we present a simpler formulation for the ride-sharing traffic equilibrium, which can be easily extended to consider different incentives for riders and drivers. Last but not least, we develop a solution approach that decomposes and reformulate the original MPEC into two network assignment problems that can be solved efficiently with existing solution algorithms.

2 METHODOLOGY

2.1 Preliminaries

Consider a congestible traffic network denoted by $G(N, A)$, where N is the set of nodes and A is the set of links. The set of origins and destinations are respectively presented by $P \subseteq N$ and $Q \subseteq N$, and we define $W \subseteq P \times Q$ as the set of origin-destination (OD) pairs. Hereafter, we will use w and $(p, q) \in W$ interchangeably to refer to an OD pair. The demand vector is defined as $\mathbf{d} = (d_w, w \in W)^\top$, where d_w denotes the total travel demand OD pair $w \in W$. We use r_{pq} (r_w) to denote a path between $w = (p, q)$ and let R_w (R_{pq}) be the set of all paths between w. Accordingly, $R = \cup_{w\in W} R_w$ gives the set of all paths of OD pairs in W. The traffic flow on path $r_w \in R_w$ is denoted by $f_{r,w}$ $(f_{r,pq})$ and thus the path flow vector is given by $\mathbf{f} = (f_{r,w}, r \in R_w, w \in W)^\top$. Similarly, we use s_a to denote the traffic flow on link $a \in A$, thus the link flow vector is $\mathbf{x} = (s_a, a \in A)^\top$. The relationship between link and path flows is represented by the link-path incidence matrix $\Delta \in 0, 1^{|A|\times|R|}$ that yields $\mathbf{x} = \Delta\mathbf{f}$. Besides, we use the OD-path incidence matrix $\Gamma \in \{0, 1\}^{|W|\times|R|}$ to present the path flow conservation $\mathbf{d} = \Gamma\mathbf{f}$. Therefore, the set of feasible path and link flows are respectively written as

$$\Omega_{\mathbf{f}}(\mathbf{d}) = \{\mathbf{f} \mid \mathbf{d} = \Gamma\mathbf{f}, \mathbf{f} \geq 0\} \tag{1}$$

$$\Omega_{\mathbf{x}}(\mathbf{d}) = \{\mathbf{x} \mid \mathbf{x} = \Delta\mathbf{f}, \mathbf{d} = \Gamma\mathbf{f}, \mathbf{f} \geq 0\} \tag{2}$$

Assume the link cost functions $\mathbf{t} = (t_a, a \in A)$ are separable, i.e., the link travel time only depends on its flow. Then, when all travelers selfishly choose paths to minimize their own travel time, the link flow pattern would converge to the well-known Wardrop's user equilibrium (UE), which is equivalent to the solution to the following VI problem (Dafermos, 1980):

$$\mathbf{t}(\mathbf{x}^*)^\top(\mathbf{x} - \mathbf{x}^*) \geq 0, \mathbf{x} \in \Omega_{\mathbf{x}}, \tag{3}$$

where $\mathbf{x}^*$ denotes the equilibrium link flows.

2.2 Formulation

We consider three traveler groups and differentiate their variables with superscripts: c for ride-sharing riders, v for ride-sharing drivers, and s for solo drivers. Particularly, we assume these groups are independent and ride-sharing drivers may or may not serve ride-sharing trips. Further, we assume each rider is served by only one driver, while drivers may serve multiple riders before reaching their destinations. However, each driver can only serve one rider at a time. Therefore, ride-sharing drivers have four types of trips: i) regular trips between their OD pairs $w \in W^v$ without serving any riders, ii) pick-up trips from their origin $p \in P^v$ or the last rider's destination $q \in Q^c$ to the next rider's origin $p \in P^c$, iii) shared trips between the rider's OD pair $w \in W^c$, and iv) drop-off trips from the last rider's destination $q \in Q^c$ to their own destination $q \in Q^v$. On the other hand, solo drivers travel between their original OD pairs $w \in W^s$. Accordingly, these trips lead to a full set of OD pairs as $\tilde{W} = W^v \cup (P^v \times P^c) \cup W^c \cup (Q^c \times Q^v) \cup (Q^c \times P^c)$.

The ride-matching decision made by the traffic manager is represented by a demand segmentation matrix $\lambda \in \Lambda := [0, 1]^{|\tilde{W}|\times|W^v|}$, where each element $\lambda_{\tilde{w},w}$ denotes the fraction of ride-sharing drivers traveling between OD pair $w \in W^v$ serving ride-sharing trips with OD pair

$\tilde{w} \in \tilde{W}$. Accordingly, all types of trips performed by ride-sharing drivers can be represented by a transformed demand vector λd^v with additional flow conservation constraints. We consider that ride-sharing drivers choose the shortest paths to deliver riders and complete their own trips. Therefore, the resulting traffic flow pattern can be derived as UE with the total demand vector $\tilde{\mathbf{d}} = M(\lambda \mathbf{d}^v) + \mathbf{d}^s$, where $M \in \{0,1\}^{|W^s||\tilde{W}|}$ consolidate virtual OD pairs to the physical network. For notation simplicity, we assume the number of physical OD pairs is the same as the number of OD pairs for solo drivers.

The Stackelberg game between the traffic manager and travelers is formulated as the following mathematical program with equilibrium constraint (MPEC):

$$\min_{\lambda \in \Lambda} \quad T(\lambda) = \mathbf{t}(\tilde{\mathbf{x}}^*)^\top \tilde{\mathbf{x}}^* + \rho^T \bar{\mathbf{d}}^c, \tag{4a}$$

$$s.t. \quad \lambda_{pq,pq} + \sum_{m \in P^c} \lambda_{pm,pq} = 1, \qquad (p,q) \in W^v, \tag{4b}$$

$$\lambda_{pm,pq} + \sum_{i \in Q^c} \lambda_{im,pq} = \sum_{j \in Q^c} \lambda_{mj,pq}, \qquad \forall m \in P^c, (p,q) \in W^v, \tag{4c}$$

$$\sum_{i \in P^c} \lambda_{im,pq} = \sum_{j \in P^c} \lambda_{mj,pq} + \lambda_{mq,pq}, \qquad \forall m \in Q^c, (p,q) \in W^v, \tag{4d}$$

$$\bar{d}^c_w = \sum_{w' \in W^v} \lambda_{w,w'} d^v_{w'} - d^c_w \geq 0, \qquad \forall w \in W^c, \tag{4e}$$

$$\tilde{\mathbf{d}} = M(\lambda \mathbf{d}) + \mathbf{d}^s, \tag{4f}$$

$$\mathbf{t}(\tilde{\mathbf{x}}^*)^T (\tilde{\mathbf{x}} - \tilde{\mathbf{x}}^*) \geq 0, \qquad \forall \tilde{\mathbf{x}} \in \Omega_{\mathbf{x}}(\tilde{\mathbf{d}}). \tag{4g}$$

The objective function $T(\lambda)$ consists of two terms: $\mathbf{t}(\tilde{\mathbf{x}}^*)^\top \tilde{\mathbf{x}}^*$ gives the total travel time over the network and $\rho^T \bar{\mathbf{d}}^c$ defines a penalty on unsatisfactory ride-matching. As per Constraint (4e), all ride-sharing riders are served though some ride-sharing drivers are detoured without serving any riders. These trips thus lack incentives and induce a penalty specified by the parameter $\rho \in \mathbb{R}_+^{|W^c|}$. Constraint (4b) dictates the conservation of departure flow from each ride-sharing driver's origin. Constraints (4c) and (4d) establish the flow conservation among the pick-up, shared, and drop-off trips for each riders' origin and destination, respectively. Finally, Constraint (4g) presents the equilibrium constraint under the transformed demand $\tilde{\mathbf{d}}$.

2.3 Solution approach

While MPEC is challenging to solve in general (Dempe, 2003), Problem (4) naturally fits into a bi-level solution procedure. At the upper level, the traffic manager updates the demand segmentation matrix λ based on the current equilibrium traffic pattern $\tilde{\mathbf{x}}^*$. The updated ride-matching decision then leads to a new transformed demand $\tilde{d}$ and further induces a shift in traffic equilibrium at the lower level. Note the lower-level problem is simply a static traffic assignment problem and can be solved efficiently via existing algorithms, e.g., the Frank-Wolfe algorithm (Frank *et al.*, 1956). The upper-level problem, however, is challenging to solve in general because of the high dimension of λ. A toy network with two driver OD pairs and two rider OD pars may end up with a matrix λ of dimension 16×2. Nevertheless, we note that the upper-level problem can be reformulated as another assignment problem over a hyper-network illustrated in Figure 1. In the hyper-network, the set of nodes consists of all origins and destinations of drivers and riders, while the links denote feasible trips. The OD pairs in the hyper-network coincide with ride-sharing drivers' OD pairs, and each path denotes a feasible trip sequence. Each element in λ then represents the link flow corresponding to each OD pair. Consequently, the upper-level problem turns into a network assignment problem with unit demand and side constraints corresponding to Eq. (4e).

The remaining question is how to connect the two levels and solve the upper-level problem in anticipation of the change in the lower-level traffic equilibrium. Note that link costs in the

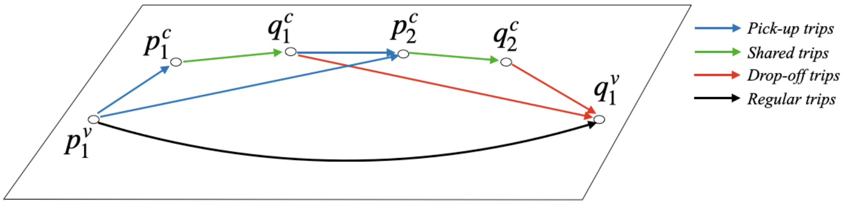

Figure 1 – *Example of a hyper-network with one driver OD pair and two rider OD pairs*

hyper-network are essentially the shortest path travel times that are readily available from the lower-level solution. We then implement the sensitivity-based approach developed by Lu (2008) to approximate the derivative of shortest path travel time with respect to demand variation. It then enables us to construct a linear approximation of the link costs in the hyper-network.

In the full paper, we will present the reformulated upper-level problem, detail the solution approach, and report the numerical experiments on toy and Sioux Falls networks. Discussions on the incentives for participating in ride-sharing will also be included.

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