A Link-based Recursive Logit Model Integrating Path-Based Attributes in Multimodal Networks

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1 INTRODUCTION

The recursive logit (RL) model has been proposed to overcome limitations of choice set generation in route choice modelling (Fosgerau et al., 2013). The model treats route choice as a sequence of link choices in transportation networks, without the need for a choice set generation process. Therefore, it can be consistently applicable for estimation and prediction. The model has been actively discussed in transportation research areas in recent years (e.g., Zimmermann et al., 2018; Oyama, 2023).

The link choice in the RL model is based on link utilities derived from link-based attributes such as link travel time and link length. These attributes are assumed to be link-additive: an attribute value for two or more links is equal to the sum of individual link attribute values, allowing link-based utilities to be expressed recursively. However, the representation of link-additive values is not straightforward for some of the key attributes describing route choice behavior in multimodal networks. The transit fare is a case in point. It is defined at the level of path and different from the simple sum of its constituent stages and varies with its composition in intermodal trips. Excluding the path-based attributes limit policy discussion of the results of the RL model in terms of willingness to pay and value of travel time savings. Moreover, considering that intermodal trips are becoming more active with emergence of smart mobility services such as ride-hailing, demand-responsive transportation, and shared mobility along with mobility-as-a-service (MaaS), this restriction will significantly limit the applicability of the RL model.

The purpose of this study is therefore to propose an extended RL model integrating the effects of path-based attributes on route choice behavior. The Moore-Penrose pseudoinverse method (Moore, 1920; Penrose, 1955) is considered to transform path-based attributes into link-based values. The applicability of the proposed model is verified through empirical analysis. We first demonstrate that the path attribute can be transformed into link-level values. Then, we show that the proposed model improves estimation and prediction results compared to the classical route choice models.

2 METHODLOGY

In a directed connected graph, the link where a traveler n is currently located is denoted by k. An RL model (Fosgerau et al., 2013) assumes that the traveler chooses the next link a that maximizes the sum of instantaneous utility $v_n(a|k)+\mu\varepsilon_n(a)$ of the link and the expected maximum downstream utility $V_n^d(a)$ recursively defined by the Bellman equation (Equation 1):

$$V_n^d(k) = \mathrm{E}\left[\max_{a\in A(k)}\left(v_n(a|k) + V_n^d(a) + \mu\varepsilon_n(a)\right)\right] \tag{1}$$

where $v_n(a|k)$ is the deterministic term of the instantaneous utility, $\varepsilon_n(a)$ is the random error term, and μ is the scale parameter.

In our proposed recursive logit with path attributes (RL-PA) model, the instantaneous utility is defined in terms of both link-based and path-based attributes. Link-based attributes are defined for each link and are link-additive, while path-based attributes cannot be directly incorporated into the instantaneous utility function and need to be transformed into link attributes. Assume that a path-based attribute is transformed into a new link attribute $x_q^l(k)$ which holds the link-additivity. Then the instantaneous utility function is defined as Equation 2:

$$v_n(k) = \sum_m \beta_m x_m(k) + \sum_q \beta_q x_q^l(k) \tag{2}$$

where $x_m(k)$ is the m^{th} link-based attribute of link k, $x_q^l(k)$ is the q^{th} transformed path-based attribute x_q^p, and β_m and β_q are coefficients. The transformation of x_q^p is expressed as follows:

$$x_q^p(\sigma) = \sum_k \delta_{\sigma k}\, x_q^l(k), \qquad for\ \forall\ \sigma, k \tag{3}$$

where $\delta_{\sigma k}$ is a binary variable that equals one if $k \in \sigma$ and zero otherwise. The exact solution for x_q^l generally does not exist because the number of unknowns does not match the number of equations. Therefore, we formulate an approximation to solve it.

$$\min \parallel \mathbf{X_q^p} - \mathbf{\Delta X_q^l} \parallel \tag{4}$$

where $\mathbf{X_q^p} = \left[x_q^p(\sigma_1) \quad \cdots \quad x_q^p(\sigma_P)\right]^{\mathrm{T}}$, $\mathbf{X_q^l} = \left[x_q^l(k_1) \quad \cdots \quad x_q^l(k_L)\right]^{\mathrm{T}}$, and $\mathbf{\Delta} = \left[\delta_{\sigma_i k_j}\right]_{1\le i\le P, 1\le j\le L}$. P and L are the number of observed paths and links. Note that $\parallel \cdots \parallel$ indicates the Euclidean norm.

This study applies the Moore-Penrose Pseudoinverse (Moore, 1920; Penrose, 1955) method to obtain the optimal solution for $\mathbf{X_q^l}$ in Equation 4 since $\mathbf{\Delta}$ does not have an inverse matrix.

$$\left(\mathbf{X_q^l}\right)^* = \mathbf{\Delta}^+\mathbf{X_q^p} \tag{5}$$

where $\left(\mathbf{X_q^l}\right)^*$ is the optimal solution for $\mathbf{X_q^l}$, and $\mathbf{\Delta}^+$ is the Moore-Penrose Pseudoinverse of $\mathbf{\Delta}$.

To further reduce the error, we also define the origin-destination specific versions of $x_q^l(k)$ and $\mathbf{X_q^l}$, which are denoted as $x_q^l(k, od)$ and $\mathbf{X_q^l(od)} = \left[x_q^l(k_1, od) \quad \cdots \quad x_q^l(k_L, od)\right]^{\mathrm{T}}$ for origin o and destination d. Let $\delta_{o\sigma}$ and $\delta_{d\sigma}$ be binary variables that equal one if the path σ originates from the origin o or is destined to the destination d, respectively, and zero otherwise. Also, let $\mathbf{\Delta_o}$ and $\mathbf{\Delta_d}$ be diagonal matrices containing $\delta_{o\sigma_i}$ and $\delta_{d\sigma_i}$ in their i^{th} entries, respectively. Then Equation 4 is specifically rewritten for origin-destination as Equation 6, and $\mathbf{X_q^l(od)}$ is obtained by Equation 7.

$$\min \parallel \mathbf{\Delta_o \Delta_d X_q^p} - \mathbf{\Delta_o \Delta_d \Delta X_q^l(od)} \parallel \tag{6}$$

$$(\mathbf{X_q^l(od)})^* = (\mathbf{\Delta_o \Delta_d \Delta})^+ \mathbf{\Delta_o \Delta_d X_q^p} \tag{7}$$

Finally, the probability of choosing a next link a conditionally on the current link k and the destination link d ($P_n^d(a|k)$) is expressed as the multinomial logit model.

$$P_n^d(a|k) = \frac{e^{\frac{1}{\mu}(v_n(a|k) + V_n^d(a))}}{\sum_{a'\in A(k)} e^{\frac{1}{\mu}(v_n(a'|k) + V_n^d(a'))}} \tag{8}$$

3 Results

3.1 Data

The methodology of this study was applied to a multimodal network and intermodal trip data in Seoul. The multimodal network comprises road, bus, and rail networks. Figure 1 shows the composition of data sets and procedures for the application. The intermodal trip data consists of two datasets. The first dataset is the National Household Travel Survey (NHTS) data, which describes the information of individual trip stages comprising a traveler's trip: departure points, arrival points, modes, departure times, arrival times, and the order of the trip stages. Since the NHTS data do not include each traveler's specific path, each trip stage of the NHTS data must be routed on a network to estimate the RL model. The second dataset, transit smartcard data, was used for routing transit (bus and rail) trip stages. Smartcard data describe transit users' boarding station and time, alighting station and time, mode type, route number (for a bus trip stage), and fare for each transit trip stage. Although it is unable to identify NHTS responses and smartcard records made by a certain traveler, it is assumed that the respondents of the NHTS data are likely to follow the same paths as recorded in smartcard data for transit trip stages with the same departure, arrival locations and times. This assumption was also used by previous studies which inferred trip purposes of smartcard records based on their corresponding NHTS responses. While transit trip stages were routed based on smartcard data, road trip stages were routed on a network using the shortest path. The paths of trip stages were then concatenated into each single-purpose trip.

3.2 Estimation Results

We compared the proposed RL-PA model against MNL and RL models. In the proposed RL-PA model, travel cost is transformed into link-level values. Table 1 reveals the estimation results. First of all, all the parameters are statistically significant at 95% significance level, and their sign is consistent with our intuition. In terms of goodness-of-fit based on AIC, the proposed RL-PA model is superior to other models. It reveals that the proposed methodology of link-based transformation effectively captures effects of path-based attributes in route choice behavior in this data set. Another advantage of the proposed model is policy discussion. For instance, the value of travel time (VOTT) based on the results of RL-PA model is 18.22 ($/hour), which is higher than that based on MNL model (17.36 $/hour). Note that VOTT based on RL model is not available since it does not consider path-based travel cost.

Based on the parameter estimates from each model, we also compare how well the models recover the actual routes used. In terms of three indices (RMSE, MAE, and MAPE), the proposed RL-PA model shows the lowest errors. It indicates that the proposed model has better prediction power than other models. Thus, we can conclude that the proposed RL-PA model successfully represents the effects of path-based attribute (travel cost) on route choice behavior.

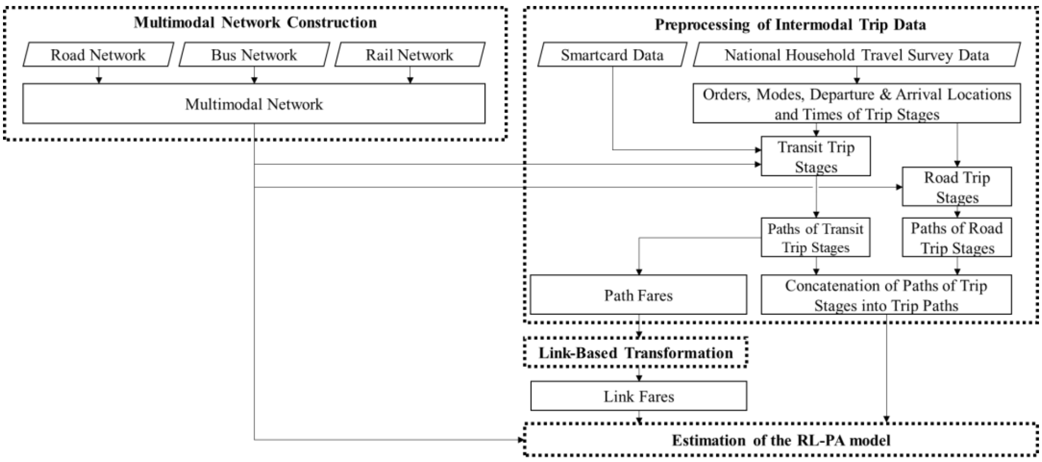

Figure 1 – *Composition of data sets and procedures for the application*

Table 1 – *Estimation Results*

	MNL	RL	RL-PA
Parameters			
Constant	-2.8930	-2.4352	-2.3356
Travel time (minutes)	-0.2100	-0.1438	-0.2110
Transfer	-5.0529	-4.7638	-4.6974
Bus link dummy	-	1.0541	1.0212
Rail link dummy	-	0.4130	0.3994
Road link dummy	-	1.2972	1.2311
Travel cost (US $*)	-0.7259	-	-0.6948
Model fit			
Final LL	-24.617	-20.036	-13.957
AIC	55.234	52.072	41.913
Model prediction			
RMSE	0.455	0.122	0.069
MAE	0.208	0.108	0.061
MAPE	55.40%	28.70%	15.47%

4 Conclusions

This study has presented the RL-PA model which enables the incorporation of path-based attributes into the RL models. We have proposed an efficient approach based on Moore-Penrose pseudoinverse method to transform path-based attributes into link-based values.

We have provided numerical results using empirical data. We use multimodal networks and observations of intermodal trips. The parameter estimates are sensible, and the proposed RL-PA model outperforms the RL and MNL models in terms of not only model fit but also prediction power. These findings reveal that our proposed method effectively extends the RL modeling framework by including path-based attributes.

Due to the limitations of space, we did not include results and discussions with link-size (LS) attribute. Consistent with previous findings, we found that the LS attribute plays an important role in the RL-PA model. We would like to present the whole results and findings at the conference.

5 References

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