Stochastic Fundamental Diagram: Calibration and Application

Cantarella G. E. [a], Cipriani E. [b], Gemma A. [b], Giannattasio O. [b], Mannini L. [b,*]

[a] Dept. of Civil Engineering, University of Salerno, Italy
g.cantarella@unisa.it
[b] Dept. of Civil, Computer Science and Aeronautical Technologies Engineering, Roma Tre University, Rome, Italy
ernesto.cipriani@uniroma3.it, andrea.gemma@uniroma3.it, orlando.giannattasio@uniroma3.it
* Corresponding author: livia.mannini@uniroma3.it

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1 INTRODUCTION

Fundamental Diagram (FD) holds relevant significance in the field of Traffic and Transportation Theory (TTT), making it a key point for on-going research. Furthermore, empirical data reveal substantial dispersion due to various factors (traffic flow conditions, users' behaviour and vehicles heterogeneity, lighting and weather); thus, the Stochastic Fundamental Diagram (S-FD) has recently emerged as a proposed solution to address these issues.

In (Siqueira, et al., 2016), a stochastic two speed state model is introduced to define FD and its variance. (Qu, et al., 2017) suggests a methodology based on percentiles for calibrating S-FD, while in (Bai, et al., 2021) the speed heterogeneity is explicitly considered. In (Wang, et al., 2021) a methodology for correcting non-realistic volumes is presented.

The present paper proposes a methodology for modelling and calibrating Stochastic Fundamental Diagram (S-FD) according to Transportation Systems Theory (TST). Specifically, starting from the methodology proposed in (Cantarella, et al., 2023), in which speed-flow functions and speed distributions (consistent with Random Utility Theory to model route choice behaviour) have been specified and calibrated from disaggregate data, the aim of the present paper is to generalise the previous approach calibrating the S-FD with respect to three different perspectives:

- with respect to different vehicle classes; variable-specific (flow and density) equivalence coefficients of vehicle classes are calibrated, required to consistently apply the S-FD;
- with respect to different data source types; parameters of the S-FD are initially calibrated with trajectory data, then the analysis and comparison between the single trajectories detected over space and time and data collected in a single section over time allows to transfer the calibrated parameters to highway sections where only portal data are available;
- with respect to different calibration methods, in order to validate the calibration results.

A real case study has been carried out in a highway context, showing promising results and verifying the effectiveness of the proposed methodology.

2 METHODOLOGY

The main definitions are outlined in the following for ease of reading. As it is well known, the main variables detected for each highway link are the flow (*f*) and the speed (*v*) that can be averaged on time or space, related with the third variable, the density (*k*), under steady-state conditions by following equation: $f = k \cdot v$.

The specification of the speed-flow function in steady state must be consistent with the appropriate travel time function for the demand assignment and transport network analysis. Moreover, the specification of speed random variable has to be reliable with the specifications of route choice behaviour models, based on disutility distributed as Normal, Gamma, Inv-Gamma random variable.

The first phase, dealing with the calibration of flow and density vehicle equivalence coefficients, is necessary to consistently apply the S-FD and, additionally, with the statistical analysis of both individual trajectories data and highway portal data. The former allows to compute spatial average speed and density. In contrast, highway portal data a concern measurement detected on a fixed section, so allowing the computation of flow and time-averaged speed. Therefore, the (spatial average) speed has been approximated by the harmonic mean of the (instantaneous) speeds, which, as confirmed by the analysis conducted in the case of the trajectories has result to be a good approximation dependent on the aggregation interval of the raw data. Additional analysis has dealt with a One-Way ANalysis Of VAriance (ANOVA) in order to compare the speed values. The findings of ANOVA indicate variations between lanes within a specific highway section, as well as differences in the central lanes between two segments due to variances in their geometry and boundary conditions.

The second phase concerns a deterministic analysis, in which several FD specifications are calibrated and compared, by means of different performance indicators and different calibration methods. The detected flows have been aggregated over a 15 to 60 second time interval and successively aggregated into flow classes with a step size of 120 vehicles per hour.

The analysed speed-density formulations have been the Greenshields' function (Greenshields, 1935), Pipes' function (Pipes, 1967), Drew's function (Drew, 1965) and Kristek's function (Kristek, 1980); moreover, the BPR-based (Bureau of Public Roads) stable regime speed-flow function. The speed-flow function can be derived from the previously cited speed-density functions for shape coefficient (b) integer solving a polynomial equation. Therefore, these functions have been analysed and calibrated by lane and by vehicle type, also calibrating the equivalence coefficients between vehicles, based on disaggregate vehicle trajectories or aggregate highway portals' data. Specifically, for the Greenshields' model, calibration has been conducted for both the free-flow speed and maximum density. Differently, besides those parameters, for Pipes' and BPR-based functions, the calibration has concerned congestion factor, shape coefficient, maximum flow.

The results of this phase show that speeds remain consistent across all models within the same aggregation level and across different aggregations; however, varying among lanes, as expected. In addition, a variation in speed between two directions of the same type of lane has been noted due to other effects, such as the presence of an on-ramp.

In addition, the speed also proved to be consistent among the different optimization methods, e.g., the non-linear gradient method and the linear regression analysis applying the least square method. Otherwise, different congestion factor and shape coefficient have resulted from the different calibrations.

Once the deterministic analysis has been carried out, the stochastic one has been performed. Specifically, the speed distribution for each flow class and lane has been studied through statistic and stochastic analyses. The detected speed frequencies have been collected in a stochastic matrix with speed in columns and flow in rows and the elements represent the (discrete) probability that the speed value belongs to a class if the flow is in specified class.

Then, for each flow class and lane the Normal, Gamma and InvGamma distributions have been fitted against the detected frequencies choosing the best distribution by means of the Kolmogorov-Smirnov test. Figure 1 shows a discretization of speed according to a probability density function InvGamma for different flow and velocity classes.

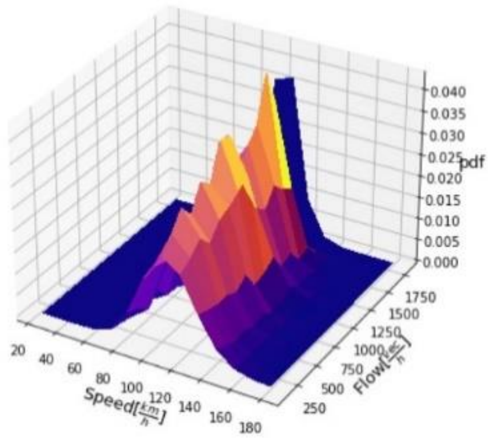

Figure 1 – *Discretization of the pdf InvGamma of speed by flow class*

3 CONCLUSION

In this paper an exhaustive methodology for specifying and calibrating Stochastic Fundamental Diagram (S-FD) is presented. Specifically, the calibration has dealt with three different perspectives: vehicle classes, data source types and calibration methods. The starting point has been the methodology proposed in (Cantarella, et al., 2023), in which speed-flow functions and speed distributions, consistent with Random Utility Theory to model route choice behaviour, have been specified and calibrated from disaggregate data. The first finding of the present paper has concerned the calibration of flow and density vehicle equivalence coefficients, required to consistently apply the S-FD. Moreover, the analysis and comparison between data from vehicle trajectories over space and time and data collected in a single section over time has been carried out, useful for huge amount of data provided by highway portals, aiming at calibrating the parameters of the speed-flow function from disaggregate data and extending them to other highway sections where only portal data are available. After these initial calibration and investigation, the second phase has concerned the deterministic analysis, in which several FD specifications are calibrated and compared, by means of different performance indicators and different calibration methods. The comparison of the parameters provided by the different methods showed that the free-flow speeds remain consistent across all models within the same aggregation level and across different aggregations and, varying among lanes, as expected. Successively, the stochastic speed analysis has been carried out, so investigating the speed distribution. Combing the results of the second and the third phases the S-FD specification is derived.

A real case study has been carried out in a highway context, showing the effectiveness of the proposed methodology. The S-FD is suitable for project assessment and evaluation, offering the capability to consider various externalities; moreover, it is useful in transportation planning. In terms of highway traffic control, the S-FD model can be exploited to predict the impacts of speed variations, such as those arising from construction zones, as well as the effects of speed control strategies like Variable Speed Limits.

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References

Bai, Lu, S. C. Wong, Pengpeng Xu, Andy H. F. Chow, and William H. K. Lam. "Calibration of stochastic link-based fundamental diagram with explicit consideration of speed heterogeneity." Transportation Research Part B: Methodological Volume 150, 2021: 524-539.

Cantarella, Giulio Erberto, Ernesto Cipriani, Andrea Gemma, Orlando Giannattasio, and Livia Mannini. "Multi-vehicle Stochastic Fundamental Diagram Consistent with Transportations Systems Theory." Procedings of 4th Symposium on Managment of Future Motorway and Urban Traffic Systems (MFTS). Dresden, 2023.

Drew, Donald R. Deterministic Aspects of Freeway Operations and Control;. Texas Transportation Institute, 1965.

Gemma, Andrea, Orlando Giannattasio, and Livia Mannini. "Motorway Traffic Emissions Estimation through Stochastic Fundamental Diagram." Sustainability 15(13), 2023: 9871.

Greenshields, Bruce D. "A study of traffic capacity." Highway research board proceedings Vol. 14. No. 1., 1935: 448-477.

Fosu, Gabriel Obed, Emmanuel Akweittey, Joseph M. Opong, and Micheal E. Otoo. "Vehicular Traffic Models for Speed-Density-Flow Relationship." Journal of Mathematical Modeling, 2020.

Kristek, R. Syntctyczny Wskaznik Jakosci Ruchu Ulicznego Jako Kryterium Sterowania. Gdańsk: Politechnika Gdańska, 1980.

Pipes, L. A. "Car following models and the fundamental diagram of road traffic." Trasportation Research UK, 1967: 21-29.

Qu, Xiaobo, Jin Zhang, and Shuanin Wang. "On the stochastic fundamental diagram for freeway traffic: Model development, analytical properties, validation, and extensive applications." Transportation Research Part B: Methodological Volume 104, 2017: 256-271.

Romanowska, Aleksandra, and Kazimierz Jamroz. "Comparison of Traffic Flow Models with Real Traffic Data Based on a Quantitative Assessment." Applied Science 11(21), 2021: 9914.

Siqueira, Adriano F., Carlos J. T. Peixoto, Chen Wu, and Wei-Liang Qian. "Effect of stochastic transition in the fundamental diagram of traffic flow." Transportation Research B: Methodology Volume 87, 2016: 1-13.

Wang, Shuaian, Xinyuan Chen, and Xiaobo Qu. "Model on empirically calibrating stochastic traffic flow fundamental diagram." Communication in Transportation Research Volume 1, 2021: 100015.