# Joint Planning of Passenger and Freight Trains in High-Speed Railway Express System

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## 1 INTRODUCTION

In recent years, high-speed railway (HSR) networks have been rapidly constructed around the world due to their highly punctual and energy-efficient nature. According to data reported by the International Union of Railways (UIC) in 2022, the operational length of high-speed railways worldwide has reached a staggering 59,498 kilometers. This extensive growth has significantly enhanced passenger convenience. However, a common problem also exists in HSR networks worldwide, where rail profitability is very low, which has a huge impact on the sustainable development of HSR construction (Zhu *[et al.](#page-4-0)*, [2023\)](#page-4-0). To address this problem, there has been a growing trend of utilizing the spare capacity of HSR for express delivery services to enhance railway utilization and improve railway profits (Sahli *[et al.](#page-4-1)*, [2022\)](#page-4-1). An efficient HSR express delivery service system benefits both the railway company and customers. However, in the HSR express system, the key problem lies in how to efficiently allocate mixed passenger-freight to the scheduling of trains, in order to achieve the optimal utilization of system capacity.

Currently, many countries are implementing express freight transportation on HSR, with China being a notable example due to its extensive operational HSR network (Yu [et al.](#page-4-2), [2021\)](#page-4-2). An example of China HSR express system is provided in Figure 1. Figure 1(a) illustrates the concept of mixed passenger-freight trains, where a part carriages can be used for loading freight. Figure 1(b) provides the scheduling of such kind of trains, where freight loading/unloading requires extra time compared to passenger trains. In practice, the Chinese rail managers employ a naive strategy for the planning of HSR express systems: they make an original train schedule without consideration of freight demand, and then adjust the plan slightly to utilize the spare capacity in the original schedule. However, this approach overlooks the potential impact of freight transport on train scheduling. Loading and unloading freight may require additional time, which can exceed the planned train stopping time in certain situations, resulting in train delays. Furthermore, this approach may not be suitable for handling an excessive amount of freight when the requirements for freight transport exceed the available train capacity. These issues can potentially lead to decreased customer satisfaction and impede the economic viability of the railway system. Therefore, it is necessary to develop an effective mathematical model that

jointly optimize the utilization and scheduling of mixed passenger-freight trains in HSR express systems.



(a) Passengers-freight trains (b) Mixed passengers-freight trains scheduling

Figure 1 – Illustration of mixed passengers-freight trains transportation system

Recently, some literature have focus on the HSR freight transport problem to fully utilize the rail capacity (Xu [et al.](#page-4-3), [2022\)](#page-4-3), [\(Zhen](#page-4-4) et al., [2023\)](#page-4-4). Give an existing train schedule, these studies aim to optimally assign passengers and freight to different trains, such that the transport profit can be maximized. The main limitation is that these studies are only suitable for handling a small volume of freight. When the number of freight items is large, adjustments to the train stopping time must be carefully considered for freight loading and unloading, which further affects the existing train schedule and may even lead to the delay of trains. To the best of our knowledge, there is limited literature that addresses the planning of mixed passenger and freight trains in HSR express systems, considering the impacts of freight loading/unloading to the train schedule.

To bridge the gap between the increased requirements in practice and the limited relevant literature, our study investigates the joint planning of mixed passenger and freight trains in HSR express systems. We develop a mixed integer linear programming (MILP) model to optimally determine the tactical train schedule, train stopping plan, and train compositions (i.e., number of passenger/freight carriages allocated to each train). The objective of our model is to minimize the total travel time and operation costs of trains while satisfying the demands of both passengers and freight. In addition, we carefully consider some new constraints in our formulation, involving the time window limitation of freight, the freight loading/unloading work time at platforms, which are practically significant while were not fully investigated in the existing literature. Then, we prove that the constructed model is an NP-hard problem, indicating its inherent difficulty in being solved. To tackle this challenge, We develop an decomposition-based approach to solve the model, where the passenger-freight allocation is formulated as the master problem, and the scheduling of each train is formulated as one subproblem. Based on the decomposition scheme, we further construct an L-shaped cut for iteratively solving the model. Real-world case studies on Wuhan-Guangzhou high-speed railway corridor are conducted to verify the effectiveness of our approach.

#### 2 Mathematical model

For the convenience of modeling, this paper introduces the following notations and variables. We use N to represent the set of stations and  $K$  to represent the set of all trains. Each train  $k \in K$  travels through stations 1, 2,  $\cdots$ , |N|. Let  $v_{k,i}^{\mathrm{u}}$  and  $v_{k,i}^{\mathrm{l}}$  denote the upper and lower bounds of running time on section i (i.e., segment between stations i and  $i + 1$ ) for each train

k. Let  $d_k^e$  and  $d_k^l$  respectively denote the earliest and latest departure times of train k from the first station, and  $h_i^{\text{da}}$  denote the minimal departure-arrival headway on the same station track to guarantee the safety separation of following trains. The acceleration and deceleration times of train k are represented by  $g_k^a$  and  $g_k^d$  respectively. The unit time required for loading and unloading freight onto a train is denoted by  $e^1$  and  $e^u$ . The loading capacity of each unit train passenger capacity and freight capacity are denoted by  $c_k$  and  $r_k$  respectively. The parameters  $w_{i,j}$  and  $z_{i,j}$  represent the number of passengers and freight requirements between station i and j respectively. The unit operation cost of each train k is denoted by  $\beta_k$ .

We next introduce the decision variables in our formulation. The decision variables in the integrated model include two types: train timetabling decision variables and passenger-freight allocation variables, which are defined as follows:

- $t_{k,i}^d$ : Departure time of train k from station i.
- $t_{k,i}^{\text{a}}$ : Arrival time of train k at station i.
- $x_{k,i}$ : Equals 1 if train k stops at station i, and 0 otherwise.
- $y_{k,l,i}$ : Equals 1 if train k departs from station i earlier than train l, and 0 otherwise.
- $m_{k,i,j}$ : The number of passengers allocated to train k between station i and station j.
- $n_{k,i,j}$ : The number of freight allocated to train k between station i and station j.
- $\mu_{k,i,q}$ : Equals 1 if train k is assigned to track q at station i, and 0 otherwise.

•  $\delta_k$ : Equals 1 if train k is a 16-composition train, and 0 if train k is an 8-composition train. The problem described above can be formulated as follows:

$$
\min \sum_{k \in K} \alpha_1 \cdot (t_{k, s_k}^a - t_{k, o_k}^d) + \sum_{k \in K} \alpha_2 \cdot \beta_k (1 + \delta_k) \tag{1}
$$

$$
\text{s.t. } d_k^e \le t_{k,o_k}^d \le d_k^1, \forall k \in K
$$
\n<sup>(2)</sup>

<span id="page-2-1"></span><span id="page-2-0"></span>
$$
\tau_i \cdot x_{k,i} \le t_{k,i}^d - t_{k,i}^a \le M \cdot x_{k,i} \quad \forall k \in k, i \in N \setminus \{o_k, s_k\}
$$
\n
$$
(3)
$$

<span id="page-2-2"></span>
$$
t_{k,i+1}^{a} - t_{k,i}^{d} \ge x_{k,i} \cdot g_{k}^{a} + x_{k,i+1} \cdot g_{k}^{d} + v_{k,i}^{1} \quad \forall k \in K, i \in N \setminus \{s_{k}\}
$$
\n
$$
\tag{4}
$$

<span id="page-2-3"></span>
$$
t_{k,i+1}^{a} - t_{k,i}^{d} \leq x_{k,i} \cdot g_{k}^{a} + x_{k,i+1} \cdot g_{k}^{d} + v_{k,i}^{u} \quad \forall k \in K, i \in N \setminus \{s_{k}\}
$$
\n
$$
\tag{5}
$$

<span id="page-2-5"></span><span id="page-2-4"></span>
$$
t_{k,i}^d - t_{l,i}^d \ge h_i^d - M \cdot (1 - y_{l,k,i}) \quad \forall k, l \in K, k \ne l, i \in N \setminus \{s_k, s_l\}
$$
  
\n
$$
t^a - t^a \ge h^a - M \cdot (1 - y_{l,k,i}) \quad \forall k, l \in K, k \ne l, i \in N \setminus \{s_k, s_l\}
$$
  
\n(6)

$$
t_{k,i}^a - t_{l,i}^a \ge h_i^a - M \cdot (1 - y_{l,k,i-1}) \quad \forall k, l \in K, k \ne l, i \in N \setminus \{o_k, o_l\}
$$
\n
$$
(7)
$$

<span id="page-2-9"></span><span id="page-2-7"></span><span id="page-2-6"></span>
$$
\sum_{q \in Q_i} \mu_{k,i,q} = 1 \quad \forall k \in K, i \in N
$$
\n
$$
(8)
$$

$$
\mu_{k,i,1} = 1 - x_{k,i} \quad \forall k \in K, i \in N
$$
\n
$$
(9)
$$

$$
t_{k,i}^{\mathbf{a}} - t_{l,i}^{\mathbf{d}} \ge h_i^{\mathbf{da}} - M \cdot (3 - y_{l,k,i} - \mu_{k,i,q} - \mu_{l,i,q}) \quad \forall k, l \in K, k \ne l, i \in N, q \in Q_i \tag{10}
$$

<span id="page-2-10"></span><span id="page-2-8"></span>
$$
\sum_{k \in K} m_{k,i,j} = w_{i,j} \quad \forall i < j, i, j \in N \tag{11}
$$

<span id="page-2-11"></span>
$$
\sum_{j \in N, j > i} m_{k,i,j} \le M \cdot x_{k,i} \quad \forall k \in K, i \in N
$$
\n
$$
(12)
$$

<span id="page-2-13"></span><span id="page-2-12"></span>
$$
\sum_{i' \in N, i' \le i} \sum_{j \in N, j > i} m_{k, i', j} \le c_k \cdot (1 + \delta_k) \quad \forall k \in K, i \in N
$$
\n
$$
(13)
$$

$$
\sum_{k \in K} n_{k,i,j} = z_{i,j} \quad \forall i < j, i, j \in N \tag{14}
$$

<span id="page-2-14"></span>
$$
\sum_{j \in N, j > i} n_{k, i, j} \le M \cdot x_{k, i} \quad \forall k \in K, i \in N \tag{15}
$$

<span id="page-2-15"></span>
$$
\sum_{i' \in N, i' \le i} \sum_{j \in N, j > i} n_{k, i', j} \le r_k \cdot (1 + \delta_k) \quad \forall k \in K, i \in N
$$
\n
$$
(16)
$$

<span id="page-3-0"></span>
$$
\sum_{j \in N, j > i} n_{k, i, j} \cdot e^l \le t_{k, i}^d - t_{k, i}^a \quad \forall k \in K, i \in N \tag{17}
$$

<span id="page-3-1"></span>
$$
\sum_{i \in N, i < j} n_{k,i,j} \cdot e^{\mathbf{u}} \le t_{k,j}^{\mathbf{d}} - t_{k,j}^{\mathbf{a}} \quad \forall k \in K, j \in N \tag{18}
$$

The objective function [\(1\)](#page-2-0) aims to minimized both the train traveling time and the train operation cost. The first five groups of constraints are basic constraints in train scheduling problem. Specifically, Constraints [\(2\)](#page-2-1) ensure that trains depart their origin station in a suitable time window to satisfy the passengers' requirements. Constraints [\(3\)](#page-2-2) are the minimum dwelling time constraints for trains which are planned to stop at stations. Constraints [\(4\)](#page-2-3)-[\(5\)](#page-2-4) are running time constraints for trains between two consecutive stations. Constraints  $(6)-(7)$  $(6)-(7)$  $(6)-(7)$  are headway constraints for trains to make each pair of trains  $k, l$  depart/arrive the same station greater than a safety time. Constraints  $(8)-(10)$  $(8)-(10)$  $(8)-(10)$  are station track related constraints to ensure trains pass through a station successfully. Specifically, Constraints  $(8)$  make sure that train k passes through station  $i$  must occupy a physical station track. In actual, main line in each station (i.e., track 1) only can be used by a trains passes it without stopping, which guaranteed by constraints [\(9\)](#page-2-9). Constraints [\(10\)](#page-2-8) ensure that two trains choose the same track satisfy a safety headway.

Constraints [\(11\)](#page-2-10) ensure that all passengers traveling between station i and j can be allocated to trains. Constraints [\(12\)](#page-2-11) ensure that when train  $k$  does not stop at station  $i$ , passengers traveling from station i to station j can not board train k. Constraints [\(13\)](#page-2-12) ensure that the number of passengers on the train remains within the maximum capacity. Constraints [\(14\)](#page-2-13) ensure that all freight transport between station  $i$  and  $j$  can be allocated to trains. Constraints [\(15\)](#page-2-14) ensure that when train k does not stop at station i, freight transport from station i to station j is not allowed to load train k. Constraints [\(16\)](#page-2-15) in the model ensure that the number of freight loads on the train does not exceed its maximum capacity. Constraints [\(17\)](#page-3-0) guarantee that there is an ample amount of time allocated for loading freight onto the train. Similarly, constraints [\(18\)](#page-3-1) ensure that there is a sufficient duration for unloading freight from the train.

#### 3 Numerical experiments

We conducted a series of numerical experiments to assess the performance of the proposed model and algorithm, utilizing data from the Wuhan-Guangzhou high-speed railway in China. The parameter settings were provided by the railway company, and passenger and freight numbers were collected from real-world data. We first tested the accuracy of the proposed method through a case study. Following that, we carried out a series of experiments using different numbers of instances to verify the effectiveness of the proposed algorithm. We compared the results of our algorithm with those obtained from the Gurobi solver to evaluate its performance. Furthermore, we designed a series of numerical experiments to verify the advantages of the proposed integration model. Finally, we analyzed the sensitivity of the model to varying numbers of passengers and freight.

#### 4 Summary

This paper constructs an optimization model to address the problem of integrating mixed passenger-freight allocation and train scheduling. To efficient solve the model, we develop a decomposition approach, where the passenger-freight allocation is formulated as the master problem, and the train scheduling is formulated as the subproblem. Based on the decomposition scheme, we further construct an L-shaped cut for iteratively solving the model. Furthermore, we apply the developed model and solution approach to a practical problem on the Wuhan-Guangzhou HSR. The results demonstrate that our model can improve rail revenue by fully utilizing the train capacity, and the proposed decomposition approach exhibits higher computational efficiency compared to the optimization solver.

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