Simulation-based network capacity allocation optimization for traffic resilience via enhanced mixed stochastic approximation

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1 INTRODUCTION

Traffic resilience represents the "ability of an urban road transportation system to prepare for different kinds of disruptions, effectively serve vehicles, and recover rapidly to its optimal serving rate" (Lu *et al.*, 2024). Compared to conventional definitions of transportation system resilience, this concept emphasizes more on traffic flow efficiency. Our previous work (Lu *et al.*, 2024) has proposed robust traffic resilience indicators based on macroscopic fundamental diagram (MFD) (Geroliminis & Daganzo, 2008) dynamics by relating system functionality, facility service rate, and trip completion rate, capable of integrating transportation network characteristics, traffic dynamics, and travel demand patterns. Moving one step forward, in this study, we are interested in the question of whether current transportation networks are with their most traffic-resilient configurations. To this end, we present a capacity allocation optimization problem with traffic resilience maximization as the objective under conditions of existing structure and infrastructure of transportation networks. In transportation networks, the capacity allocation problem can be formulated as a combination of lane allocations in two-way roads and traffic signal timing. Since lane allocations and traffic signal timing are typically modeled as integer and continuous variables, respectively, this problem can be formulated as a Mixed Network Design Problem (MNDP).

Given the efficacy of traffic simulations in modeling the sophisticated interplay between transportation supply and demand and in estimating detailed traffic states under various decision strategies, we propose a simulation-based approach to addressing the problem discussed above. Specifically, a capacity allocation is sought, such that the resulting transportation network exhibits the "best" performance according to network-wide traffic resilience indicators. A stochastic approximation (SA) coupled with simultaneous perturbation (SP) gradient approximation is utilized to locate the minimizer of the traffic simulator SUMO where only noisy evaluation is available. In particular, we integrate recent advances in second-order SA algorithms (Spall, 2009) and mixed SP implementations (Wang *et al.*, 2018) to develop a novel second-order mixed simultaneous perturbation stochastic approximation (2MSPSA) algorithm to solve the simulation-based optimization problem.

2 METHODOLOGY

2.1 Traffic resilience indicator based on macroscopic fundamental diagrams

The MFD-based indicators proposed in Lu *et al.* (2024) for measuring traffic resilience to congestion and supply disruptions is employed here. They evaluate the functionality of transportation systems with the trip completion rate derived from MFDs, such that the resilience loss can be calculated by integrating the reduction in trip completion rates over the entire disruption period, which is expressed as

$$R^d = -\int_{t_0}^{t_1} (D_c - D(t))\, H(k(t) - k_c)\ \mathrm{d}t \tag{1}$$

$$R^s = -\int_{t_0}^{t_1} \max\{D^s(t) - D(t), 0\}\ \mathrm{d}t \tag{2}$$

where t_0 and t_1 indicate the start and end of the disruption. $D(t)$ and $D^s(t)$ denote, at time t, the trip completion rates under normal and disruptive conditions, respectively. D_c denotes the critical trip completion rate under normal conditions. R^d and R^s represent the resilience to traffic congestion and supply disruptions (infrastructure malfunction), respectively. $H(\cdot)$ is a Heaviside step function, indicating whether traffic is in congestion states.

2.2 Capacity allocation

Denote the design capacity of a lane as q^{cap}. Note that the effective capacity is different from its design value, depending on the control measures in effect. For a certain approach of an intersection, the effective capacity of a lane belonging to this approach can be estimated by $q^{\mathrm{eff}} = q^{\mathrm{cap}} t^g / T$ (see Figure 1), where t^g is the green time assigned to the phase of this approach and T is the signal cycle. Simply speaking, the capacity of a link is determined by its number of lanes and the traffic signal plan. It follows that capacity allocation is achieved by altering lane allocations and traffic signals. Therefore, lane allocation and traffic signal schedule are the two types of decisions employed in this study to perform transportation network capacity allocation.

2.3 Simulation-based capacity allocation optimization

The simulation-based capacity allocation optimization (SOCA) problem can be expressed as

$$\begin{aligned}
\text{(SOCA)} \min_{\boldsymbol{z},\boldsymbol{x}} \quad & \mathbb{E}_{\boldsymbol{\xi}} = [y(\boldsymbol{z}, \boldsymbol{x}, \boldsymbol{\xi})] && (3)\\
\text{s.t.} \quad & \mathbb{E}_{\boldsymbol{\xi}} = [g(\boldsymbol{z}, \boldsymbol{x}, \boldsymbol{\xi})] \le 0 && (4)\\
& \boldsymbol{z} + \boldsymbol{z}^{\mathrm{opp}} = \boldsymbol{n} && (5)\\
& \boldsymbol{B}\boldsymbol{x} = \boldsymbol{T} && (6)\\
& \boldsymbol{z}_l \le \boldsymbol{z} \le \boldsymbol{z}_u && (7)\\
& \boldsymbol{x}_l \le \boldsymbol{x} \le \boldsymbol{x}_u && (8)\\
& \boldsymbol{z} \in \mathbb{Z}^d, \boldsymbol{x} \in \mathbb{R}^{p-d} && (9)
\end{aligned}$$

where $\boldsymbol{z}$ denotes discrete lane allocation decisions and $\boldsymbol{x}$ denotes continuous traffic signal decisions. Function $y(\cdot)$ represents the absolute traffic resilience loss with uncertainty which can be evaluated through traffic simulation for a particular instance of decision inputs $\boldsymbol{\theta} = [\boldsymbol{z}^\top, \boldsymbol{x}^\top]^\top$ and a realization of the random variables in the simulation, i.e., the vector $\boldsymbol{\xi}$. $g(\cdot)$ is a vector-valued function representing the constraints that need to be evaluated during the simulation, such as route choice probabilities. Due to the complementary relationship between the lane numbers of two links in opposite directions, the effective decisions in $\boldsymbol{z}$ reduce to half. This complementary constraint can be expressed as Equation (5). The effective decisions in $\boldsymbol{x}$ are also fewer than the length of $\boldsymbol{x}$ with the consideration of a fixed traffic signal cycle. Equation (6) represents the complementary relationship between phase splits belonging to the same traffic signal cycle, where $\boldsymbol{B}$ is a signal-phase matrix describing the subordination of phases to traffic signals, and $\boldsymbol{T}$ denotes the cycle time of these signals. In addition, bound constraints are imposed on the decision variables. $\boldsymbol{z}_l$ and $\boldsymbol{z}_u$ in Equation (7) denote the possible minimum and maximum number of lanes of links, respectively. One can specify $\boldsymbol{z}_l = \mathbf{1}_{n_l}$ where $\mathbf{1}_{n_l}$ is a vector of ones with length n_l, the total number of links of the network, to avoid eliminating any existing routes. Similarly, $\boldsymbol{x}_l$ and $\boldsymbol{x}_u$ in Equation (8) denote the minimum and maximum phase splits of traffic signals, respectively. Note that in this formulation, the stochastic functions are considered with their expected values.

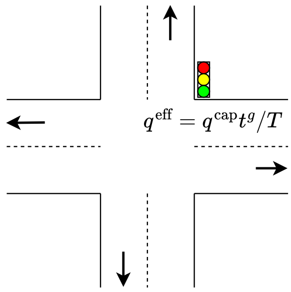


Figure 1 – *Calculation of effective capacity.*

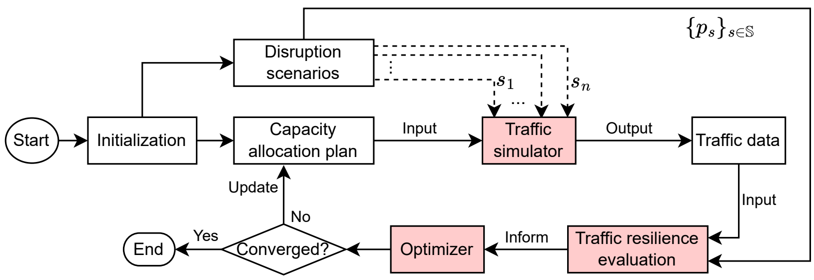


Figure 2 – *Flowchart of simulation-based resilient network design based on capacity allocation.*

In the case of traffic resilient network design, the objective function Equation (3) will be converted to

$$y(\boldsymbol{z}, \boldsymbol{x}, \boldsymbol{\xi}) = -w^n D_c - w^d R^d(\boldsymbol{z}, \boldsymbol{x}, \boldsymbol{\xi}) - \sum_{\phi \in \mathbb{S}} w^s_\phi R^s(\boldsymbol{z}, \boldsymbol{x}, \boldsymbol{\xi}, \phi) \tag{10}$$

where $\mathbb{S}$ indicates the set of typical supply disruption scenarios in the given city, and ϕ indicates a certain disruption. w^n, w^d and w^s_ϕ indicates, in capacity allocation optimization, the weights on the resilience to normal daily operations, hyper-congestion and supply disruptions, respectively.

Figure 2 presents the framework for simulation-based resilient network design optimization.

2.4 Second-order mixed simultaneous perturbation stochastic approximation

Consider the simulation-based optimization setup, where $R : \mathbb{Z}^d \times \mathbb{R}^{(p-d)} \mapsto \mathbb{R}$ is the resilience loss function to be minimized. The noisy loss function evaluation for $\boldsymbol{\theta}$ is then given by $y(\boldsymbol{\theta}) = R(\boldsymbol{\theta}) + \epsilon(\boldsymbol{\theta})$ where $\epsilon(\cdot)$ represents a noise function. Second-order SA algorithms incorporate the following updating recursion

$$\hat{\boldsymbol{\theta}}_{k+1} = \hat{\boldsymbol{\theta}}_k - a_k \bar{\bar{\boldsymbol{H}}}_k^{-1}(\hat{\boldsymbol{\theta}}_k) \hat{\boldsymbol{G}}_k(\hat{\boldsymbol{\theta}}_k) \tag{11}$$

where $k = 0, 1, \ldots, N$ indicates the iteration index, $\{a_k\}_{k\geq 0}$ is a positive decaying scalar gain sequence, $\hat{\boldsymbol{G}}_k(\hat{\boldsymbol{\theta}}_k)$ is the approximated gradient, and $\bar{\bar{\boldsymbol{H}}}_k$ denotes the approximation of the Hessian information. SP methods approximates the gradient by

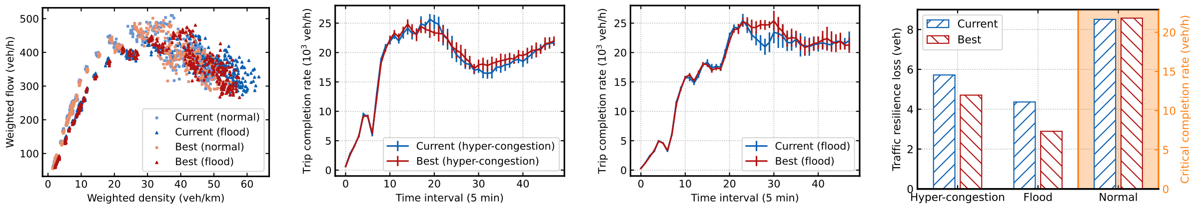
$$\hat{\boldsymbol{G}}_k(\hat{\boldsymbol{\theta}}_k) = \frac{y_k(\hat{\boldsymbol{\theta}}_k^{(+)}) - y_k(\hat{\boldsymbol{\theta}}_k^{(-)})}{2\boldsymbol{C}_k \circ \boldsymbol{\Delta}_k} \tag{12}$$

where $\hat{\boldsymbol{\theta}}_k^{(\pm)} = \boldsymbol{\pi}(\hat{\boldsymbol{\theta}}_k) \pm \boldsymbol{C}_k \circ \boldsymbol{\Delta}_k$ with $\boldsymbol{\pi}(\hat{\boldsymbol{\theta}}_k) = (\lfloor \hat{\boldsymbol{\theta}}_{k,1} \rfloor + 0.5, \ldots, \lfloor \hat{\boldsymbol{\theta}}_{k,d} \rfloor + 0.5, \hat{\boldsymbol{\theta}}_{k,d+1}, \ldots, \hat{\boldsymbol{\theta}}_{k,p})^\top$, $\boldsymbol{\Delta}_k$ represents a p–dimensional vector with each element independently generated from a Bernoulli ± 1 distribution with probability 0.5 for each outcome, and $\boldsymbol{C}_k = (0.5, \ldots, 0.5, c_k, \ldots, c_k)^\top$ with the first d elements as 0.5 and the rest are c_k, where $\{c_k\}_{k\geq 0}$ is also a positive decaying scalar gain sequence. For the Hessian estimate $\bar{\bar{\boldsymbol{H}}}_k$, we extend the one presented in Spall (2009) to the problems with mixed variables. More details will be provided in the full version of the paper.

3 RESULTS

We implement a case study using the network of Munich city center, Germany, which is about 100 km^2 large with 2605 links. A calibrated mesoscopic SUMO model is used to simulate the traffic dynamics under different network states for the morning period from 5 am to 10 am. We consider a network-wide flood disruption scenario and model it as a 30% speed reduction due to road inundation. In the SOCA objective, weights are set as 0.6, 0.2, and 0.2 for normal (w^n), hyper-congestion (w^d), and flood (w^s) operation conditions, respectively.

Figure 3a shows that the influence of the capacity allocation plan on the shape of MFD is minor, while the impact of supply disruptions (flood as an example) is significant. We compare the trip completion rates with the current network state and the "best" capacity allocation state found by the algorithm in Figure 3b and Figure 3c for hyper-congestion due to extreme demand and global urban flood scenarios, respectively. The best allocation plan leads to a more stable trip completion rate under both scenarios compared to the current network state. Figure 3d depicts an overall comparison of these two capacity allocation plans. The best capacity allocation plan improves the critical trip completion rate (normal operations) by 0.7%, and mitigate the traffic resilience loss to hyper-congestion and flood by 17.7% and 33.7%, respectively.



(a) *MFD comparison* (b) *Hyper-congestion* (c) *Flooding* (d) *Loss comparison*

Figure 3 – *Traffic dynamics and resilience comparisons between current and "best" allocations.*

4 DISCUSSION

We constructed a simulation-based capacity allocation optimization (SOCA) problem to investigate if the existing network structure of the transportation system can be better configured to make it more resilient to traffic jams during daily operations, unexpected extreme demand, and supply disruptions. SOCA can be easily adapted to different urban contexts by changing the weights on different disruption scenarios accordingly. We presented a 2MSPSA algorithm to solve this kind of MNDP. The preliminary results showed that the algorithm can effectively address the SOCA problem, and transportation networks indeed can be more resilient by improving the network capacity allocation.

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