Controlling Automated Vehicles on Lane-free Roundabouts via a Nonlinear Controller

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1 INTRODUCTION

Roundabouts play a substantial role in urban traffic, facilitating enhanced efficiency at light traffic while potentially becoming a bottleneck during peak periods (Flannery and Datta 1997). Hence, the proficient management of roundabouts can contribute to the enhancement of traffic flow in the surrounding area; however, they present challenges because of their geometrical complexity and frequent major conflicts among vehicles. Several methodologies are developed to guide autonomous vehicles driving on roundabouts, see a brief review in (Naderi *et. al*, 2024). Most of the existing works are lane-based, although large roundabouts may operate without lanes and should be considered as lane-free infrastructures.

Naderi *et. al*, (2024) proposed a generic control strategy for automated vehicles on large lane-free roundabouts where a nonlinear controller, designed for ring-roads, along with linear boundary controllers, aiming to keep vehicles within introduced corridors, were employed. Despite the effectiveness of the presented approach, it is worth trying different approaches that may provide benefits in terms of simplicity or efficiency. In this paper, we employ a new nonlinear controller, designed by Karafyllis *et al.* (2023), that integrates some elements of the previous approach into the feedback law. In particular, it is designed for curvy boundaries that facilitates incorporating roundabout and Origin-Destination (OD) corridors. Also, it adaptively changes the vehicles' desired speed in an integrated way. On the other hand, the new feedback law has also some limitations that do not allow its direct implementation for the case of roundabouts and necessitates some modifications, which are briefly discussed in the rest of this paper, along with some preliminary results.

2 VEHICLE DYNAMICS AND FEEDBACK LAW

2.1 Kinematic bicycle model

In view of frequent and major turnings while driving on roundabouts, the kinematic bicycle model is employed to represent vehicle kinematics, with state equations (Karafyllis *et al.* 2023):

$$\begin{aligned}
\dot{x} &= v\cos(\theta) \\
\dot{y} &= v\sin(\theta) \\
\dot{\theta} &= \sigma^{-1} v\tan(\delta) \\
\dot{v} &= F
\end{aligned} \tag{1}$$

where x and y are the longitudinal and lateral position coordinates of the rear axle midpoint of the vehicle, v is vehicle speed, and θ is its orientation with respect to the fixed x-axis, which is typically taken to point to the general movement direction; while the y-axis is perpendicular to it, and the origin of the coordinates may be placed at some point in the middle of the road. Acceleration F and steering angle δ are two control inputs of the model. Finally, σ is the length of the vehicle.

2.2 Nonlinear Feedback Control (NLFC)

Karafyllis *et al.* (2023) developed a decentralized two-dimensional nonlinear cruise controller for vehicles driving on roads with curvy boundaries that guarantees some important features such as collision-free movement, remaining within boundaries, and tracking of (different) desired speeds for all vehicles. Importantly, every vehicle may have its own (curvy) boundaries of its movement corridor, which may or may not coincide with the road. The feedback law reads

$$\delta_i = \tan^{-1}(\sigma_i u_i / v_i), \quad i = 1,\ldots,n \tag{2}$$

$$F_i = \frac{1}{Q\big(v_i,\theta_i, f_i(w)\big)}\left(-\mu_1(v_i\cos(\theta_i) - f_i(w)) + \frac{Z_i(w) + v_i\sin(\theta_i)u_i}{v_i(v_{\max} - v_i)} - \Phi_i(w) - g_i(x_i,y_i)\Xi_i(w)\right)$$

where

$$u_i = \frac{2\big(\cos(\theta_i) - \cos(\Theta)\big)^2}{Rh_i(x_i,y_i,\theta_i)}\left(\begin{array}{l} -\mu_2 v_i^2\big(\sin(\theta_i) - g_i(x_i,y_i)\cos(\theta_i)\big) + R\dfrac{v_i\cos(\theta_i)a_i(x_i,y_i,\theta_i)}{\cos(\theta_i)-\cos(\Theta)} \\ -U_i'\left(\dfrac{2y_i - \big(\beta_i(x_i) + \alpha_i(x_i)\big)}{\beta_i(x_i) - \alpha_i(x_i)}\right)\dfrac{2v_i}{\beta_i(x_i)-\alpha_i(x_i)} - v_i\Xi_i(w)\end{array}\right)$$

$$g_i(x,y) := \frac{\big(\beta_i(x) - y\big)\alpha_i'(x) + \big(y - \alpha_i(x)\big)\beta_i'(x)}{\beta_i(x) - \alpha_i(x)}$$

$$f_i(w) = v_i^* b\big(\Phi_i(w) + g_i(x,y)\Xi_i(w)\big)$$

$$\Phi_i(w) = \sum_{j\neq i} V_{i,j}'(d_{i,j})\big(x_i - x_j\big)/d_{i,j}$$

$$\Xi_i(w) = \sum_{j\neq i} p_{i,j}V_{i,j}'(d_{i,j})\big(y_i - y_j\big)/d_{i,j}$$

$$h_i(x,y,\theta) = 2\cos(\theta)\big(\cos(\theta)-\cos(\Theta)\big) + \sin^2(\theta) + \sin(\theta)g_i(x,y)\big(\big(\cos(\theta) - 2\cos(\Theta)\big)\big)$$

$$\begin{aligned} a_i(x_i,y_i,\theta_i) &= \cos(\theta_i)\left(\frac{y-\alpha_i(x_i)}{\beta_i(x_i)-\alpha_i(x_i)}\beta_i''(x_i) + \left(1 - \frac{y_i - \alpha_i(x_i)}{\beta_i(x_i)-\alpha_i(x_i)}\right)\alpha_i''(x_i)\right) \\ &\quad + \big((\beta_i(x_i) - \alpha_i(x_i))\sin(\theta_i) - g_i(x_i,y_i)\cos(\theta_i)\big)\frac{\beta_i'(x_i) - \alpha_i'(x_i)}{\big(\beta_i(x_i)-\alpha_i(x_i)\big)^2}\end{aligned}$$

$$Q(v,\theta,l) = \frac{v_{\max}v\cos(\theta) + lv_{\max} - 2lv}{2v^2(v_{\max} - v_i)^2} \tag{3}$$

$$b(x) = \begin{cases} 1 & x \le \varepsilon \\ \exp(-(x-\varepsilon)^3) & x > \varepsilon \end{cases}$$

where $v_{\max}$ is the maximum allowable speed, R, μ_1, μ_2, and ε are positive design parameters. Also, Θ is the maximum allowable orientation, i.e. θ must remain in the range of $(-\Theta,\Theta)$. In particular, $\Phi_i(w)$ and $\Xi_i(w)$ reflect longitudinal and lateral repulsions, respectively; $f_i(w)$ is the adaptive desired speed depending on the impact of neighboring vehicles that indirectly takes the density of the surrounding area into account; $\alpha_i(x)$ and $\beta_i(x)$ introduce the right and left corridor boundaries of vehicle i which should be sufficiently smooth. Moreover, the vehicle-repulsive and boundary-repulsive potential functions are:

$$V_i(d)=\begin{cases} q\dfrac{(\lambda-d)^3}{d-L} & L<d\le\lambda \\ 0 & d>\lambda \end{cases}$$
$$U_i(y)=\begin{cases} \left(\dfrac{1}{1-y^2}-c\right)^2 & y\in\left(-1,-\sqrt{\dfrac{c-1}{c}}\right)\cup\left(\sqrt{\dfrac{c-1}{c}},1\right) \\ 0 & y\in\left[-\sqrt{\dfrac{c-1}{c}},\sqrt{\dfrac{c-1}{c}}\right] \end{cases} \tag{5}$$

where $c\ge 1$, $q>0$, $\lambda>L>0$ are design parameters, The inter-vehicle elliptic distance is defined by:

$$d_{i,j}:=\sqrt{\left(x_i-x_j\right)^2+p_{i,j}\left(y_i-y_j\right)^2} \tag{6}$$

where $p_{i,j}=p_{j,i}\ge 1$ determines the shape of elliptic aura around the vehicle. $i,j=1,\dots,n$.

3 USING NLFC FOR VEHICLES ON ROUNDABOUTS

The feedback law, presented in Section 2, cannot be directly employed for controlling vehicles on roundabouts due to the following reasons:

1- The vehicle orientation must be in the range $(-\Theta,\Theta)$ where $0<\Theta<\pi/2$. Clearly, vehicles exceed the orientation limit while driving on the entire area of a roundabout i.e., their orientation will be between $[-\pi,\pi]$.

2- The speed-tracking term appears as $(v_i\cos(\theta_i)-f_i(w))$ where $f_i(w)$ is the adaptive speed. Hence, a vehicle follows the desired speed correctly only if $\theta_i=0$.

To address these difficulties, we introduce new moving coordinates for each vehicle, whose x'-axis is aligned with the roundabout's circular angle at the current position; and the perpendicular y'-axis points towards the roundabout centre. Also, the middle of the circular road is considered as the origin of coordinates, see. Fig. 1. Thus, in the new coordinates, the longitudinal position of ego vehicle is always zero, while the lateral position is its radial distance from the middle of the circular road. The latter has a radius $R_a=(R_{in}+R_{out})/2$. Finally, θ' is the orientation with respect to the x'-axis or, in other words, the deviation from the circular angle at the current position of the vehicle. Consequently, the relative longitudinal and lateral distance between ego and adjacent vehicles, which appear in the feedback law, must be calculated in the new coordinates. The roundabout's inner and outer circular boundaries must also be expressed as the left and right boundaries, $\alpha(x')$ and $\beta(x')$, in new coordinates. As illustrated in Fig. 2, they can be determined by projecting circles into the coordinates of each vehicle:

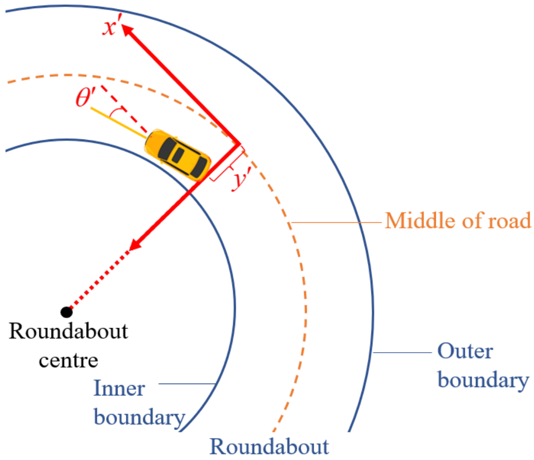

Figure 1 – *Moving coordinates of each vehicle*

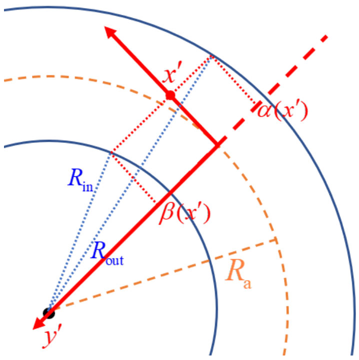

Figure 2 – *Left and right boundaries in the transformed coordinates*

$$\alpha(x') = R_a - \sqrt{R_{out}^2 - x'^2}$$
$$\beta(x') = R_a - \sqrt{R_{in}^2 - x'^2} \quad (5)$$

With these changes, the control inputs of ego vehicle can be calculated by the feedback law, i.e. (2), while replacing $x_i, y_i, \theta_i, x_j, y_j$ with the transformed variables $x'_i, y'_i, \theta'_i, x'_j, y'_j$, respectively.

The presented strategy is successfully implemented for vehicles rotating on a well-known case study, the Charles de Gaulle roundabout in Paris, whose inner and outer radii are respectively 46 m and 84 m. A video of the preliminary simulation results can be seen at https://bit.ly/49ZPfEJ where 100 vehicles with different initial conditions and desired speeds are smoothly moving on the roundabout without colliding with each other or exceeding boundaries, while reaching the respective desired speeds.

Next step of this research focuses on definition on entrance and exit scenarios which will be done through defining appropriate OD corridors.

References

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