Optimal Curbside Pricing and Space Allocation for Managing Multi-modal Travelers in Dynamic Networks

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1 INTRODUCTION

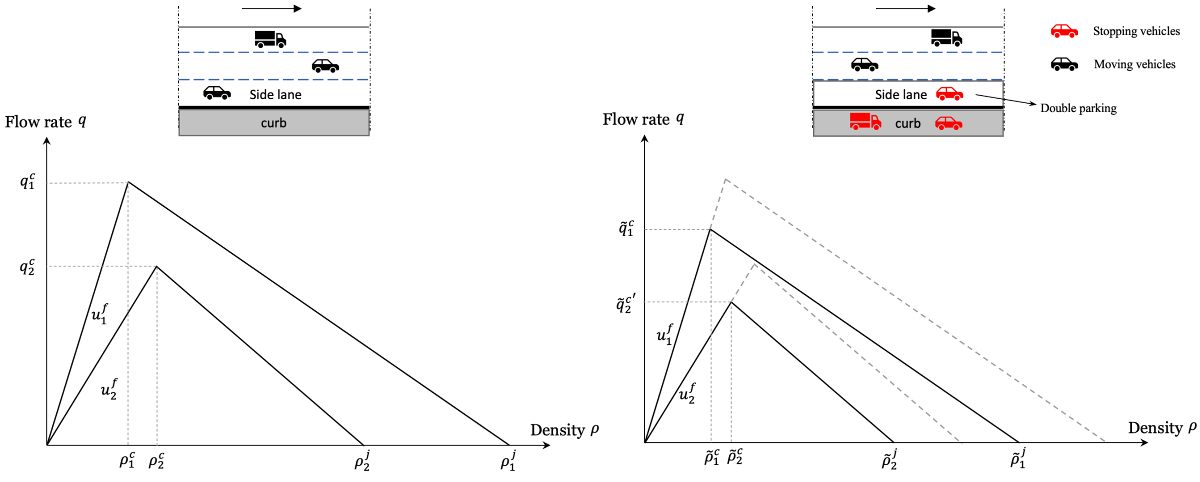
Curb spaces has been increasingly critical in recent years due to the widespread e-commerce with on-demand delivery(Xu & Sun, 2024) and expansion of ride-sourcing services (Liu *et al.*, 2023a). The intensive usage of the limited public resources by multi-modal travelers without adequate regulation leads to competition between various users and illegal parking behaviors. This situation is exacerbating the congestion not only on curbs but also the broader transportation system. Some existing research investigates the curb usage pattern and develops targeted pricing and space allocation strategies (Liu *et al.*, 2023a, Lim & Masoud, n.d., Maxner *et al.*, 2023, Liu *et al.*, 2023b), however, the research gap lies on designing optimal control policy for curb spaces in dynamic network models which integrates both multi-modal travelers' behavior patterns (i.e., route choice, curb choice) and spatio-temporal network flow dynamics. To this end, this study proposes a framework to design optimal curbside pricing and space allocation policies for managing multi-modal curb users in general dynamic networks. Our framework combines curb-aware flow dynamic of multi-class traffic and travelers' route/curb choice behavior in mesoscopic modeling with the optimization of curbside operations.

2 FORMULATION

In this study, we consider three curb users, represented by three travel modes: private cars using curb spaces for on-street parking (denoted by $\mathcal{D}$), ride-hailing vehicles using curbs for passenger pick-up and drop-off (denoted by $\mathcal{R}$), and commercial trucks using curbs for loading/unloading or deliver packages. For each curb user, we defined its trip travel mainly consisting of two parts: (1) cost of experienced travel time and (2) cost of curb use. The experienced travel time depends on the spatio-temporal traffic conditions and the curb use cost depends on curb user type, curb space availability and arrival time. For private car drivers, the travel cost of trip k is formulated as

$$c^{rs}_{\mathcal{D},k,t} = \max[\gamma(t + w^{rs}_{\mathcal{D},k,t} - t^{*,rs}_{\mathcal{D}}), \beta(t^{*,rs}_{\mathcal{D}} - t - w^{rs}_{\mathcal{D},k,t})] + \alpha w^{rs}_{\mathcal{D},k,t} + p^{k}_{\mathcal{D}}(o^{a_k}_{t'}),\ \forall k \in P^{rs}_{\mathcal{D}} \tag{1}$$

where the first term is schedule delay penalty function for modeling departure time choice which depends on the the difference between actual arrival time $t + w^{rs}_{i,k,t}$ and desired arrival time



Original FD (w/o parking)

Truncated FD (w/ parking)

Figure 1 – *Curb-aware Fundamental Diagram*

$t_i^{*,rs}, i \in \mathcal{C} = \{\mathcal{D}, \mathcal{R}, \mathcal{T}\}$. $\alpha w^{rs}_{\mathcal{D},k,t}$ denotes the travel cost regarding the travel time and $p^k_{\mathcal{D}}(o^{a_k}_{t'})$ denotes the curb usage fee depending on the curb occupancy and fee rate of the destination link a_k of trip k at arrival time t'. We decompose the full service traces for ride-hailing and delivery trucks into sequential trips since the time-varying demand and path set are assumed to be known. Hence the travel cost for a ride-hailing trip $\forall k \in P^{rs}_{\mathcal{R}}$ is

$$c^{rs}_{\mathcal{R},k,t} = \max[\gamma(t+w^{rs}_{\mathcal{R},k,t}-t^{*,rs}_{\mathcal{R}}), \beta(t^{*,rs}_{\mathcal{R}}-t-w^{rs}_{\mathcal{R},k,t})]+\alpha w^{rs}_{\mathcal{R},k,t}+\rho(d^{rs}_{\mathcal{R},k}, w^{rs}_{\mathcal{R},k,t})+p^k_{\mathcal{R}}(o^{a_k}_{t'}),\ \forall k \in P^{rs}_{\mathcal{R}} \tag{2}$$

where $\rho(d^{rs}_{\mathcal{R},k}, w^{rs}_{\mathcal{R},k,t})$ is the service fee term for ride-hailings. Similarly, for the decomposed delivery trip $\forall k \in P^{rs}_{\mathcal{T}}$, the travel cost is

$$c^{rs}_{\mathcal{T},k,t} = \max[\gamma(t + w^{rs}_{\mathcal{T},k,t} - t^{*,rs}_{\mathcal{T}}), \beta(t^{*,rs}_{\mathcal{T}} - t - w^{rs}_{\mathcal{T},k,t})] + \alpha w^{rs}_{\mathcal{T},k,t} + p^k_{\mathcal{T}}(o^{a_k}_{t'}),\ \forall k \in P^{rs}_{\mathcal{T}} \tag{3}$$

In this study, the mesoscopic dynamic network loading is used to model the spatio-temporal network flow and travelers' route/curb choice responding to traffic conditions and curb regulations, and the resultant conditions are used to compute travel costs for multi-modal travelers which govern their choice behavior. In the curb-aware DNL, the fundamental diagrams of links are functions of exogenous curb utilization along the road segment, meaning that the fundamental diagram for different locations of the same road may not be identical and depends on the specific location-based curb utilization condition (i.e., curb occupancy), shown in Figure 1.

To model travelers' route/curb choice, the en-route route/curb choice behavior is integrated into the DNL which can approximate the instantaneous equilibrium flow pattern. To broaden the curb locations for a user to choose from, we design a virtual destination (VD) layer in the road network connecting several curb spaces around, shown in Figure 2. For a curb user following en-route choice policy, he/she can use one VD as destination to search for the shortest path and choose the curb location connected to the VD when he/she arrives. This setting can give travelers more choices for curb usages within a region.

We formulate the optimal curb pricing and space allocation separately as two mathematical program problems, each with corresponding decision variables. For the optimal curb pricing (OCP) problem, the objective function is the total travel cost for all travelers of three modes with decision variables of curb prices and path flows, defined in Equation 17. The constraints are (1) dynamic network loading function which uses path flows as input and outputs the traffic conditions (mainly link flow $x^a_{i,t}$ and link travel time $h^a_{i,t}$ in this study) in Equation 18; (2) behavior model Ψ which regulates travelers' route/curb choices based on traffic and curb conditions, defined in Equation 19; (3) fixed time-varying demand of curb usage for multi-modal travelers

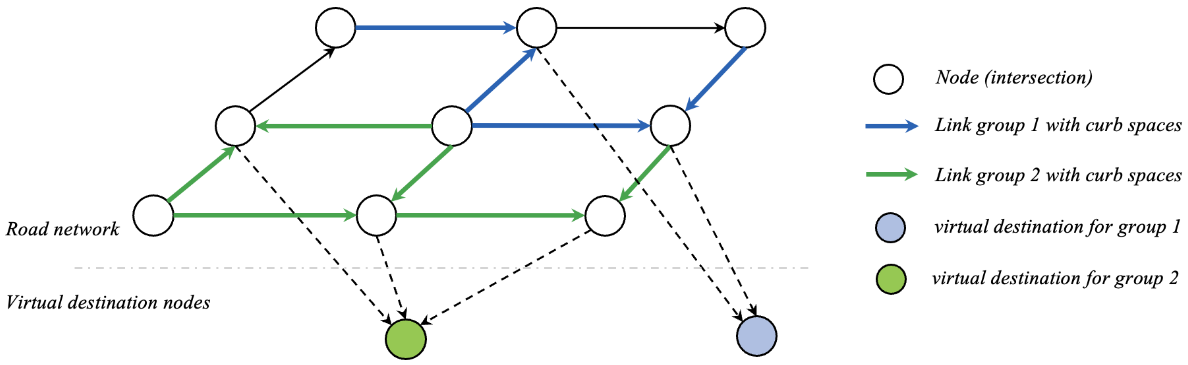


Figure 2 – *Virtual destinations designed for en-route curb choices*

and non-negative path flow, defined in Equation 20 and 21 respectively; (4) upper and lower bounds of time-varying curb usage prices for different users in Equation 9.

$$\mathrm{TTC}_{p^a_{i,t}, f^{rs}_{i,k,t}} = \sum_i \sum_t \sum_{r,s} \sum_k c^{rs}_{i,k,t} \cdot f^{rs}_{i,k,t} \tag{4}$$

$$\text{s.t.} \quad \{h^a_{i,t}\}_{a,i,t} = \Lambda\left(\{f^{rs}_{i,k,t}\}_{rs,i,k,t}\right) \tag{5}$$

$$\{f^{rs}_{i,k,t}\}_{rs,i,k,t} = \Psi\left(c^{rs}_{i,k,t}, \{h^a_{i,t}\}_{a,i,t}, \{p^a_{i,t}\}_{a,i,t}\right) \tag{6}$$

$$q^{rs}_{i,t} = \sum_k f^{rs}_{i,k,t} \tag{7}$$

$$f^{rs}_{i,k,t} \geq 0 \tag{8}$$

$$\underline{p}^a_{i,t} \leq p^a_{i,t} \leq \bar{p}^a_{i,t} \tag{9}$$

Similarly, the optimal curb space allocation model can be formulated with the same the objective function which is total travel cost of all curb users but the decision variables are curb capacities for different curb user types. The constraints also have behavior model, dynamic network loading function, fixed time-varying demand and non-negative path flows. The only difference is the constraint of summation of curb capacities cannot exceed the total spaces available for curb parking.

$$\mathrm{TTC}_{\kappa^a_{i,t}, f^{rs}_{i,k,t}} = \sum_i \sum_t \sum_{r,s} \sum_k c^{rs}_{i,k,t} \cdot f^{rs}_{i,k,t} \tag{10}$$

$$\text{s.t.} \quad \{h^a_{i,t}, x^a_{i,t}\}_{a,i,t} = \Lambda\left(\{f^{rs}_{i,k,t}\}_{rs,i,k,t}\right) \tag{11}$$

$$\{f^{rs}_{i,k,t}\}_{rs,i,k,t} = \Psi\left(c^{rs}_{i,k,t}, \{h^a_{i,t}\}_{a,i,t}, \left\{p^a_{i,t}\left(x^a_{i,t}/\kappa^a_{i,t}\right)\right\}_{a,i,t}\right) \tag{12}$$

$$q^{rs}_{i,t} = \sum_k f^{rs}_{i,k,t} \tag{13}$$

$$f^{rs}_{i,k,t} \geq 0 \tag{14}$$

$$C^a \geq \sum_i \kappa^a_{i,t} \tag{15}$$

The behavior model in this study is incorporated into the mesoscopic dynamic network loading which approximates the equilibrium traffic flows. Both formulations can be seen as mathematical programming problems with equilibrium constraints (MPEC). Additionally we introduce an upper-level objective aimed at maximizing total revenue from curb management (Equation 16-

22), which is of significant interest to public agencies.

$$\mathrm{TR}_{p^a_{i,t}, f^{rs}_{i,k,t}} = \sum_i \sum_t \sum_{r,s} \sum_k f^{rs}_{i,k,t} \cdot p^a_{i,t} \tag{16}$$

$$\text{s.t.} \quad \{f^{rs}_{i,k,t}\}_{rs,i,k,t} \in \arg\min \mathrm{TTC} = \sum_i \sum_t \sum_{r,s} \sum_k c^{rs}_{i,k,t} \cdot f^{rs}_{i,k,t} \tag{17}$$

$$\{h^a_{i,t}\}_{a,i,t} = \Lambda\left(\{f^{rs}_{i,k,t}\}_{rs,i,k,t}\right) \tag{18}$$

$$\{f^{rs}_{i,k,t}\}_{rs,i,k,t} = \Psi\left(c^{rs}_{i,k,t}, \{h^a_{i,t}\}_{a,i,t}, \{p^a_{i,t}\}_{a,i,t}\right) \tag{19}$$

$$q^{rs}_{i,t} = \sum_k f^{rs}_{i,k,t} \tag{20}$$

$$f^{rs}_{i,k,t} \geq 0 \tag{21}$$

$$\underline{p}^a_{i,t} \leq p^a_{i,t} \leq \bar{p}^a_{i,t} \tag{22}$$

3 EXPERIMENTS

Due to the complexity of the dynamic network loading function and the multi-level optimization structure, the problem is solved heuristically based on dynamic system optimum (SO) solution. The numerical experiments will be conducting on two networks: one small toy network consisting 18 links and a larger Pittsburgh Downtown network covering 91 main neighbourhoods. The dynamic network model of Pittsburgh Downtown region is calibrated by real-world multi-source data including curb monitoring data.

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