Mean Field Game in Autonomous Lane-Free Traffic

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1 INTRODUCTION

Connected and Autonomous Vehicles (CAVs) are poised to revamp the planning and operation of future urban transportation systems. Envisioning this future, (Johansen and Løvland, 2015) visualized highways without traditional unidirectional lanes, with CAVs freely moving across the highway's two-dimensional space, a concept later termed as Lane Free Traffic (LFT) by Papageorgiou et al. (2021). Recent literature on LFT CAV navigation algorithms demonstrates its potential to increase highway capacity under various conditions (Johansen & Løvland, 2015; Yanumula et al., 2023; Dabestani et al., 2023). Yet, most existing algorithms primary focus on optimizing the navigation of individual CAVs within the LFT system. Overarching LFT issues such as system wide traffic management, interactions among multiple vehicles, and bottleneck dissipation remain largely unaddressed. Designing the interactions among multiple CAVs on the LFT can be engineered to collectively optimize traffic flow on the freeway system.

This paper addresses the above challenges through a Mean Field Game (MFG) framework, taking significant steps in this direction. MFG optimizes the individual actions of a multitude of CAVs while capturing the overall impact on the traffic system (Lasry and Lions, 2007). This focus on individual system's dynamic is instrumental in the success of LFT systems especially in busy traffic conditions., where individual decisions made by each CAVs—whether speed adjustment or lateral movement—are rather intertwined, collectively permeating the state of the traffic dynamics.

This paper makes significant methodological and theoretical contributions in the field. Methodologically, it pioneers the application of MFG to LFT systems, aiming to optimize system traffic flow. This objective is accomplished by finely regulating the interactions among numerous CAVs while also accounting for the individual trip characteristics of occupants. Theoretically, this work advances our understanding of dynamic vehicle interactions within transportation systems. It develops a comprehensive framework that shapes future traffic management strategies and guides decision-making for next-generation transportation infrastructure planning.

The rest is as follows: Section 2 describes the methodology of using MFG for CAVs interactions in LFT. Section 3 presents preliminary results. Section 4 concludes by summarizing key points.

2 METHODOLOGY

In developing navigation algorithms for CAVs within LFT environments, the principle of locality becomes critical as CAV's interactions are limited to nearby agents, rather than the entire system's population. Yet, to effectively manage traffic on freeways, it is essential to consider the behavior and interactions of all vehicles within the system. A MFG approach can effectively consider large populations of CAVs without losing sight for both micro and macro scale considerations. Influenced by the aggregate state of other CAV agents (Huang et al., 2006; Lasry and Lions, 2007), each CAV is modeled through interactions with its immediate neighbors using fields of influence (Bayraktar et al., 2022). As depicted in Figures 1a and 1b, each CAV's field of influence graph represents interaction degrees with adjacent CAVs, fostering navigation algorithms that realistically address the localized nature of vehicle interactions in LFT scenarios and enhance individual driving strategies by considering both detailed and collective behaviors.

For instance, vehicle i in Figure 1a, highlighted in yellow, employs an MFG game framework, summarized in Figure 1c, to derive its navigation algorithm's control parameters in terms of acceleration and steering angle. In this game, there are two players, one is the CAV and the other is the field of influence around it. Each CAV i makes a decision (i.e. acceleration and angle) that attempts to balances its individual utilities with the collective one. The field of influence around this subject CAV adjusts its density/flow in response. The process continues until an equilibrium is reached.

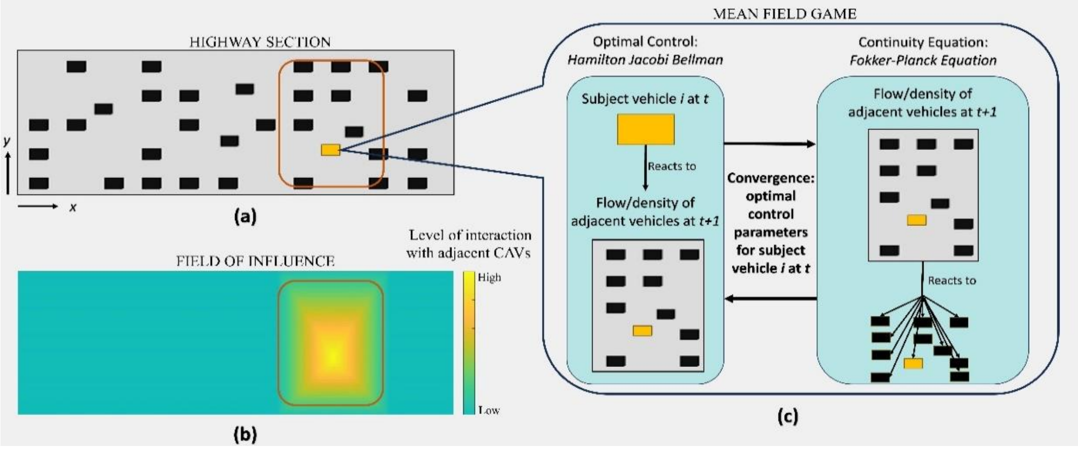

Figure 1 – *(a) section of the highway with subject vehicle in yellow, (b) field of influence diagram showing the interaction level of subject vehicle with its surroundings, (c) MFG flowchart.*

The MFG based framework outlined next harmonizes individual strategies with the broader dynamics of traffic flow.

Individual Cost Functions and Control Decisions: Each CAV operates based on a disutility (i.e. cost) function, G_i (Eq 1), which aims to meet several driving sub-objectives. Each of these sub-objectives is weighted by a factor (w), allowing the vehicle to alter its two control decisions: acceleration (a) and steering angle (θ). Instances of the sub-objectives include:

1. **Speed Alignment:** Vehicles calculate the running cost of deviating from a desired speed ($v_{desired}$) during the control interval *T*. This involves adjusting acceleration to either catch up or slow down to this target speed, ensuring efficiency and compliance with speed regulations.
2. **Steering Consistency:** Steering angles are optimized to meet the vehicle's directional targets while also aligning with the average steering angles of nearby vehicles ($\bar{\theta}_{vic}$) during the control interval *T*. This prevents abrupt or inconsistent lateral movements, promoting smoother traffic.
3. **Traffic Density Matching:** Vehicles adjust their position and speed to match the local surrounding field's vehicle density (ρ_i) to an optimal distribution ($\rho_{optimal}$). The aim is to avoid congestion and optimize the spatial allocation of CAVs on the available freeway space. A higher-level controller strategically allocates these CAVs to maximize traffic throughput, taking trip priorities into account (e.g. CAVs with higher occupancies, emergency vehicles, et.).

$$G_i(x,y,t) = \min_{\substack{a_i:[t,T]\to[0,a_{max}] \\ \theta_i:[t,T]\to[\theta_{min},\theta_{max}]}} \left[\int_t^T \left[w_1 |v_i(s) - v_{desired}| + w_2 \left[\left(\theta_i(s) - \bar{\theta}_{vic}(s) \right) \right] \right] ds + w_3 \, |\rho_i(x,y,T) - \rho_{optimal}(x,y,T)| \right] \quad (1)$$

Collective Optimization through MFG Framework (Figure 1c): The MFG framework uses the following cost elements in a game-theoretic model to optimize traffic at both micro and macro levels:

1. **Hamilton-Jacobi-Bellman (HJB) Equation:** The dynamic equation **(Eq 2)** optimizes acceleration and steering angles of CAV in real-time, to meet the speed alignment and steering consistency sub-objectives, while also incorporating vehicle's technical dynamics constraint. Thus, $\lambda_x.a.\cos(\theta)$ and $\lambda_y.a.\sin(\theta)$ in the HJB equation, enable the vehicle to calculate how different values of acceleration and steering angles affect its trajectory and positioning in the pursuit of optimal reduction of the total system cost. λ_x and λ_y are Lagrange multipliers.

$$\frac{dG}{dt} + \min_{a,\theta}\{w_1|v - v_{desired}| + w_2(\theta - \bar{\theta}_{vic}) + \lambda_x.a.\cos(\theta) + \lambda_y.a.\sin(\theta)\} = 0 \quad (2)$$

2. **Fokker-Planck Continuity Equation:** Complementing the HJB, equation **(Eq 3)** evaluates the impact of individual CAV decisions on the overall traffic flow. In Eq 3, (v_x, v_y) represents the velocity field of the vehicles in the x (longitudinal)- and y (lateral)-directions, respectively. $\nabla.\left(\rho(x, y, k)\ (v_x, v_y)\right)$ is the divergence of the product of density and velocity, capturing how the change of vehicle flow in the field affects its vehicles distribution. The equality equation is a classical flow conservation equation that entails that the local change in density over time is balanced by the spatial change in flow (i.e. into or out of the field of influence as in Figure 1b).

$$\frac{\partial \varphi}{\partial t} + \nabla.\left(\varphi(x, y, k)\ (v_x, v_y)\right) = 0 \quad (3)$$

Strategic Interaction and Tradeoffs:

1. **Individual vs. Collective Benefit:** Each vehicle strikes a balance between optimizing its own journey and contributing to efficient traffic flow. Based on the MFG framework, a CAV might occasionally sacrifice individual preference (e.g. reaching an ideal speed or a desired lateral position) for greater overall traffic efficiency.
2. **Local Interaction vs. Global Impact:** While each CAV primarily interacts with nearby vehicles, the collective behavior shaped by these local interactions influences traffic patterns across the entire system. This highlights the importance of each vehicle's role, no matter how seemingly insignificant, within the larger network.

Dynamic Adaptation and System-Wide Equilibrium: Through continuous computation and adaptation based on the HJB and Fokker-Planck equations, CAVs strive to achieve a near Nash equilibrium—a state where no vehicle can reduce its cost without increasing another's. This equilibrium is not static but dynamically adjusts as traffic conditions evolve, reflecting a complex interplay between individual actions and collective outcomes.

In essence, CAVs on a busy freeway play a complex, collaborative game where each vehicle's decisions are tightly interwoven with those of others, collectively pursuing a harmonious and efficient traffic system. This traffic gamification enhances travel experiences for individual commuters while also improving the overall performance of the transportation system.

3 NUMERICAL RESULTS AND DISCUSSION

To assess the performance of the MFG-based LFT traffic management framework, we conducted a MATLAB simulation on a 5 km, 10 m wide hypothetical freeway segment. Without traditional lanes delineation, the freeway has a free-flow speed of 100 km/h and an on-ramp and an off-ramp at 1 km and 2 km distances respectively. The simulation lasts 70 minutes with a 10-minute warm-up phase and is replicated with five runs using different random seeds. The loaded demand on the main highway and entry ramp varies by 20 minutes interval to mimic peak hour fluctuations, with exit ramp traffic constituting 10% of the main flow. The preliminary results of the average of the 5 runs are summarized in Figure 3. The computational time for each simulation run, conducted on a Core i9 processor, averaged approximately 43 minutes.

Figure 3a and Figure 3b show that during the initial 20 minutes of the simulation, inflow rates are low and accordingly outflow rates closely match the inflow, indicating a balanced traffic condition with no significant delays or congestion; maintaining optimal travel speeds.
However, in the second 20-minute interval, inflow rates increase, creating congestion. The outflow rates surge and yet they don't match the inflow, indicating a reduced discharge capacity. This capacity drop leads to congestion formation, subsequently degrading freeway performance.

During the simulation's final phase, starting at the 40-minute mark, inflow demand decreases and outflows initially exceed inflows, clearing congestion within 5 to 6 minutes. This critical phase restores free-flow conditions, ultimately establishing a steady state where the demand is once again met by capacity, allowing vehicles to travel at optimal speeds without delays.

Figures 3b and Figure 3c show the results of the time-space diagram for 2 scenarios: the first allows unrestricted lateral vehicle movements, while the second restricts these movements to a frequency of one lateral movement per 1.5 minute. Figure 3c exhibits less homogeneity and a notable decline in speed in the weaving zones associated with on-ramps and off-ramps. This phenomenon is primarily ascribed to the imposed restrictions on vehicles' lateral positions. As a result, vehicles are compelled to remain behind slower-moving traffic or navigate through congested zones. In contrast,

Figure 3b, representing the MFG model for LFT with no restrictions on lateral maneuvers, demonstrates a more efficient traffic flow management, particularly evident in its ability to mitigate congestion and maintain smoother traffic conditions, especially around critical weaving areas.

In evaluating the safety of the LFT model, the implementation and analysis of the Time to Collision (TTC) metric plays a pivotal role. Drawing on the work of Nadimi et al., (2020), who developed a new strategy for calculating TTC in the context of angular collisions, the safety analysis within our LFT model adopts this approach as a framework for safety evaluation. Since the average TTC does not confer many insights about the traffic safety conditions, a surrogate safety measure, collision probability, is used to evaluate the safety condition by comparing the calculated TTC and the threshold TTC. The collision probabilities reported for the two scenarios presented are 0.261 and 0.087, respectively, corresponding to a threshold TTC of 3 sec and 2 sec.

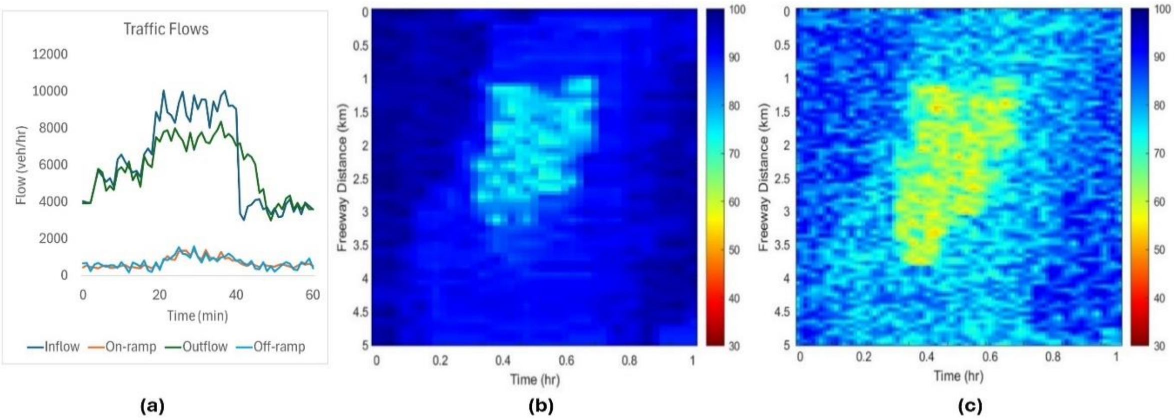

Figure 2 – *(a) Simulation traffic flows; (b) Spatio-temporal diagram of MFG lane free traffic scenario; Spatio-temporal diagram of MFG with restricted lateral movement scenario*

4 CONCLUSION

In conclusion, the developed MFG based model formulates a novel dual traffic management/navigation algorithm for CAVs travelling in LFT. One of the unique features of MFG framework is its “zoom” capability, which adeptly maps the state of each CAV and its direct field of influence, to the overall state of the system. To put it differently, individual CAVs driving decisions. while minimally compromising their self-driving objectives, are engineered to achieve a collective harmonious traffic flow that is efficient and safe. These micro-macro traffic management strategies are instrumental parts of future traffic operations, facilitating the transition from traditional lane-based to a fluid-like LFT; thereby paving the way for transformative changes in how we operate, manage, and plan our future transportation infrastructure.

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