Flow-Dependent Facility Location Optimization in Continuous Space for Bike-share Network Design

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1 INTRODUCTION

Bike-sharing offers a sustainable way of mobility commonly used as a standalone active mode of transportation or to enhance connectivity to other modes, like transit, through first- and last-mile solutions. Bike-share systems often start as small-scale pilot projects and expand gradually in response to growing user demand. By optimizing the design of bike-share systems, mainly through strategic station placement, the continued growth of the network becomes more impactful.

Determining the optimal location of bike-share stations presents a challenging task in designing an effective network. It involves more than just geography and requires a detailed knowledge of decision variables that impact the network's functionality, including the number and location of stations, station's capacity, and the allocation of bicycles. The placement of bike-share stations goes beyond providing accessibility; it involves analyzing bike flow and mobility patterns. We can identify the most frequented routes by analyzing origin-destination (OD) trip data and strategically placing stations along these paths to optimize usage. This strategic placement, guided by OD trip analytics, is the key to enhancing the bike-share network's efficacy, making each station a dynamic hub that adapts to the ever-changing mobility landscape.

Research on bike-sharing location optimization highlights diverse strategies for station placement, focusing primarily on accessibility (Frade & Ribeiro (2015)) while often overlooking the network effects of ridership that necessitate advanced algorithmic approaches like Continuum Approximation (CA). CA methods are used in terminal designs (Ouyang & Daganzo, 2006), transit network design (Daganzo (2010), Ouyang *et al.* (2014)), optimizing bike-share systems (Caicedo *et al.*, 2023), and integrating transit and bike-share networks (Luo *et al.*, 2021).

This study introduces a CA model aimed at optimizing the placement of bike-share stations to maximize ridership. The algorithm strategically adjusts each station's location within a continuous service area, where demand is spatially distributed, to align with OD patterns and maximize network efficiency. This proposed methodology prioritizes ridership, unlike previous studies focusing primarily on coverage. Numerical experiments demonstrate the effectiveness of the CA algorithm, delivering near-optimal solutions that enhance bike-sharing ridership.

2 Methodology

2.1 Problem Statement

This study addresses the problem of locating a set of N bike-share stations located at $X = \{x_1, x_2, \ldots, x_N\}$ within a continuous service area S to maximize the total ridership. The origins and destinations are characterized by spatial density functions $\Lambda_I(x)$ and $\Lambda_J(x)$, respectively, for $x \in S$. Since bike-sharing services often operate in homogeneously-populated and developed urban areas, we assume origins and destinations are independently distributed. The influence area of station i, denoted by A_i is defined as the region that encloses the origin and destination points closest to station i than any other station. The areas cannot become indefinitely large because bike-share users are only willing to walk a certain distance (or time) to access each station. We define this accessibility constraint on the system by ensuring the maximum influence area is $A_{\max}$. We define the total trip production and termination from/to station i is defined as

$$P_i(X) = \int_{x \in A_i} \Lambda_I(x), \tag{1}$$

and

$$T_i(X) = \int_{x \in A_i} \Lambda_J(x), \tag{2}$$

respectively. We define the ridership between the station pair (i, j) by $\lambda_{ij}\Big(P_i, T_j, C_{ij}(x_i, x_j)\Big)$, which depends on the ridership production and termination, and the impedance factor, $C_{ij}(x_i, x_j)$ which depends on the station locations and their distance from one another specifically. We later further explain the properties and empirical validation of the impedance function in the study.

We present the following mathematical problem to maximize the total ridership:

$$\text{Maximize}_X \quad \sum_{i=1}^{N}\sum_{j=1}^{N} \lambda_{ij}\Big(P_i, T_j, C_{ij}(x_i, x_j)\Big) \tag{3}$$

$$s.t \quad x_i \in S, \quad \forall i \tag{4}$$

$$A_i \subseteq S, \quad \forall i \tag{5}$$

$$A_i \cap A_j = \emptyset, \quad \forall i \neq j \tag{6}$$

$$|A_i| \leq A_{\max}, \quad \forall i \tag{7}$$

where the first two constraints ensure the stations and the influence areas are enclosed within the service areas, respectively, the third constraint ensures the influence areas are mutually exclusive, and the fourth constraint ensures the area of each influence area is less than the allowable limit. We note that the final constraint stipulates that the regions are not necessarily collectively exhaustive, especially when $A_{\max} N \ll |S|$.

2.2 Algorithm

We use a force-based mechanism within a continuous service area S. The algorithm initiates by defining each influence area, A_i, of station i by a “disk" (similar to Ouyang & Daganzo (2006)) of area $A_{\max}$. This area is subject to three distinct repulsive or attraction forces influencing its optimal location. We assume S is connected, which permits the free movement of a station and its disk anywhere within S without intersecting the boundary. The forces are detailed below:

Station-to-Station Force ($F_{SS_{ij}}$): The relationship between trip frequency and station distance exhibits a log-normal distribution, as demonstrated by Toronto bike share data in Figure 1-a. This trend also appears in many other micro-mobility (bike-sharing specifically) services and has a logical justification in that when the trip distance is too short, the users may prefer to walk, and when it is too long, they may prefer a motorized mode. Hence, there is an optimal inter-station distance, D_{opt}, that maximizes trip frequency. $F_{SS_{ij}}$ exerts a repulsion between stations closer than D_{opt} and an attraction for those farther apart to encourage an even distribution that reflects maximum trip frequency. The force is applied to capture the impedance relationship explained earlier in the objective function (3). The station-to-station force is mathematically formalized as $F_{SS_{ij}} = P_i T_j \left(\frac{1}{\exp\left(-\frac{(\ln(d_{ij}) - \ln(D_{opt}))^2}{2\sigma^2}\right)} - 1\right) \cdot \text{sgn}(d_{ij} - D_{opt})$, where d_{ij}

denotes the distance between two stations, P_i and T_j are respective production and termination of station i and j (equations (1) and (2)), and σ is the standard deviation of the distribution. The term $\text{sgn}(d_{ij} - D_{opt})$ determines the sign of the force vector, negative (repulsion) or positive (attraction). The force is designed to move stations towards the optimal spacing, as illustrated in Figure 1-b.

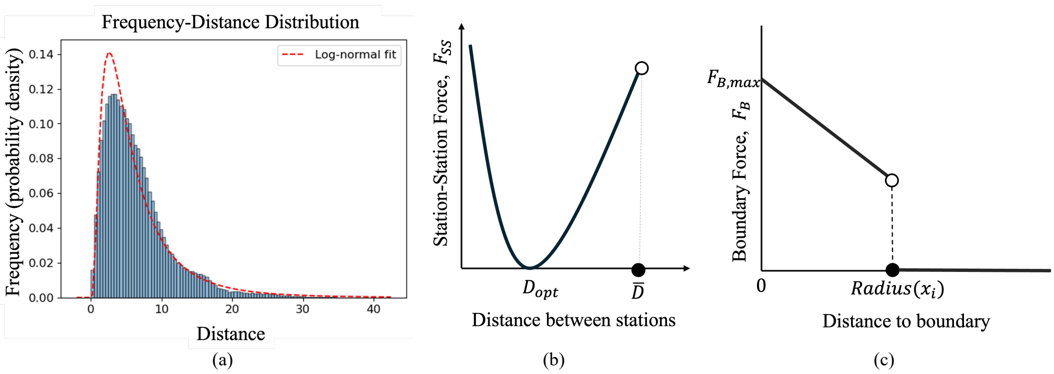

Figure 1 – *a)Trip Frequency-Distance Distribution, b)Station-Station Force, c)Boundary Force.*

Origin and Destination Force (F_{Oi}/F_{Di}): The origin-destination (OD) densities, $\Lambda_I(x)$ and $\Lambda_J(x)$, apply attraction forces F_{Oi} and F_{Di}, which seek to increase the trip production and terminations of (1) and (2). The forces are formalized as $F_{Oi} = \frac{P_O \cdot Q_i^\beta}{d_{Oi}^\alpha}$ and $F_{Di} = \frac{T_D \cdot Q_i^\beta}{d_{Di}^\alpha}$, where d_{Oi} and d_{Di} are the distances between origin and destination to station i, respectively. P_O and T_D are respective origin and destination demands, and Q_i represents the station's capacity. The exponents α and β will be determined through empirical analysis.

Boundary Force (F_B): The repulsive boundary force F_B occurs when the circumference of a station's disk intersects with the boundary of S, ensuring the station's influence area remains within the service area (similar to what explained in Ouyang & Daganzo (2006)). This force increases linearly as the station approaches the boundary (Figure 1-c).

The optimization algorithm iteratively refines station placements by assessing station production and termination and calculating resultant forces. A doubly constrained gravity model guides the process, aiming to maximize overall system ridership.

3 Computational Example

3.1 Data

We conducted a computational example for ten bike-share stations to assess the algorithm's performance. We divide the service area into cells, each assigned specific origin and destination demand values derived from density functions (similar to that explained in Li *et al.* (2016)).

3.2 Model Development

We conceptualize each bike-share station's disk and cap the station-to-station force, limiting their influence to a maximum range derived from data distribution. Calculating the resultant force for each station, the model uses small step sizes for disk movements, denoted by μ, and assume an adaptive step sizes that gradually decrease with the iterations, to promote swift convergence. Station adjustments trigger recalculation and updating of the P_i and T_i values, reflecting the coverage of their influence areas. A doubly constrained gravity model then calculates trip distributions and total ridership. If maximum ridership is not achieved, the model recalculates the forces, relocates stations, and recalculates trip distributions and ridership, iterating this process until an optimal configuration that maximizes overall ridership is identified.

4 Discussion

This study introduces a continuum approximation algorithm for locating bike-share stations aimed at ridership maximization. The algorithm adjusts each station's location based on evaluated forces until optimal position is found. Unlike previous methods that mainly focused on coverage, the proposed methodology prioritizes ridership by ensuring that the stations are placed in locations that attract users and support them in completing their trips efficiently. The numerical example validates the effectiveness of the CA algorithm. Figure 2-a illustrates the forces that impact each station. In the beginning, as shown in Figure 2-b, stations move significantly while searching for spots that meet the optimal distance and fulfill the demands attraction. The Voronoi tessellation (Figure 2-c) helps investigate the changes in each station's area over various iterations. The observed movement patterns through the iterations show that optimal station placement is a complex balance of inter-connectivity, geographical centrality, and proximity to high-demand areas, ensuring each station enhances network cohesion. The next step of our study is to incorporate station capacity into the optimization model and introduce a multi-criteria objective function that emphasizes top-priority criteria such as equity and transit integration. Additionally, the expanded numerical case study in the final paper will focus on the city of Toronto.

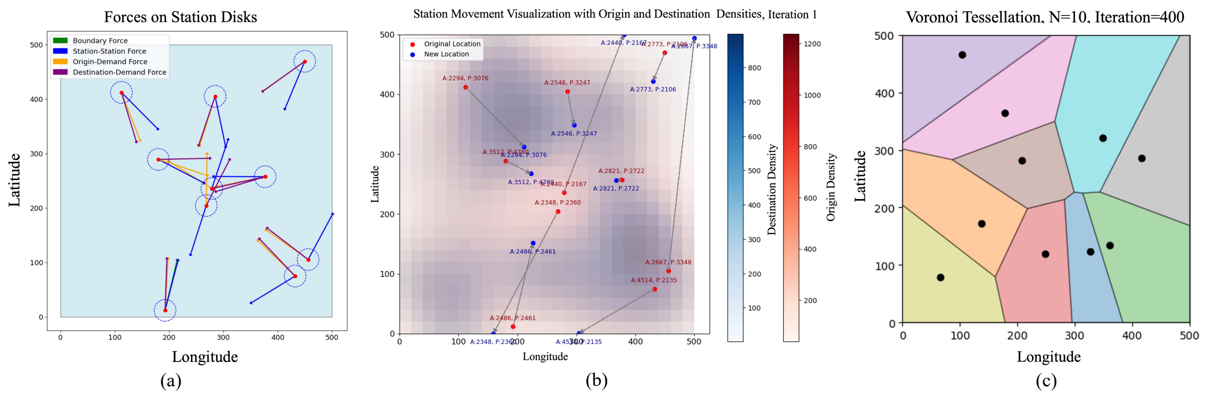

Figure 2 – *a)Forces on each station, b)Stations relocation, c)Voronoi tessellation.*

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