Optimizing timetable schedules for non-traffic hour maintenance window of urban rail transit systems

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1 INTRODUCTION

Urban rail transit (URT) systems serve as the backbone of public transportation systems in cities, and regular maintenance work is vital for the service safety, efficiency, and reliability. Preventive maintenance is usually performed during the non-traffic hours (NTHs). In Hong Kong, the current operational schedule on mass transit railway (MTR) allows for only limited NTHs, approximately 4–5 hours each night. However, out of this timeframe, only around 2 hours are available for actual maintenance tasks; as the remaining time is needed for necessary preparations and post-work site clearance. As the need for NTH possession increases due to maintenance, asset upkeep/renewal and new project works, the Hong Kong MTR Corporation Limited is proactively exploring the possibility of adjusting the train service's start and end times to allow for longer NTH times. This should be pursued after all other options to increase NTH have been exhausted, and the impact on passengers should be considered (MTR Corporation Limited, 2023). In this paper, we jointly investigates the train frequency setting and timetabling during late-night and early-morning periods under different requirement for NTH maintenance window. The objective of the optimization includes maximizing the operation profits, minimizing the total passenger waiting time, and minimizing the negative impact of losing demand (i.e., the unserved passenger demand who intended to travel outside the service period). The contribution of this work is two-folded. First, we provide closed-form solutions to represent optimal schedules, including the optimal number of train services and their headways during late-night and early-morning periods. Second, we analyze the coordination between the two periods to meet an evolving requirement for NTH maintenance window.

2 METHODOLOGY

As shown in Figure 1, this study focuses on optimizing the train timetables on a uni-directional URT line during late-night and early-morning hours to accommodate the required NTH maintenance window. We denote some parameters, variables and sets in our analysis, these include:

observation horizon $[0,\Delta]$, length of late-night (early-morning) service period Δ^n (Δ^m), the unit operation cost per train service C_{nc}, the unit service cost for serving each passenger trip C_{sc}, the value of platform waiting time VOT, the proportion of unserved passengers who reschedule their trip timing to catch the last night/first morning train service $\hat{\zeta}/\bar{\zeta}$, the average ticket fare of night/morning services per passenger trip $\hat{f}/\bar{f}$, the total number of passenger demand during night/morning $\hat{\theta}/\bar{\theta}$, the scale parameter of the night/morning passenger demand distribution $\hat{\lambda}/\bar{\lambda}$, the required NTH maintenance window Δ^{NTH}, the train headway between train $i-1$ and i at night $\hat{H}_i$, the train headway between train i and $i-1$ in the morning $\bar{H}_i$, the departure time of train i at night/in the morning $\hat{T}_i/\bar{T}_i$, the number of night/morning train services N/M, the set of all night train indexes $S_n = \{1, 2, \ldots, N\}$, and the set of all morning train indexes $S_m = \{1, 2, \ldots, M\}$. We optimize the number of late-night train services (N) and their headways $\hat{H}_j$, $j \in S_n$, and the number of early-morning train services (M) and their headways $\bar{H}_k$, $k \in S_m$, and determine $\hat{T}_N$ and $\bar{T}_M$ accordingly. The available NTH window, which lasts from $\hat{T}_N$ to $\bar{T}_M$, should be sufficient to meet the required time. Coordination between the late-night and early-morning train services is critical to fulfill the NTH maintenance window requirement, and passenger demand is a leading element in this optimization. We summarize our assumptions for this study as follows: late-night passenger demand decreases monotonically with time, while early-morning passenger demand increases monotonically with time. Passenger arrival rate is consistent across all stations, but each station has a time delay relative to the starting station, which is equivalent to the travel time of the train from the starting station to that station. Additionally, a proportion of unserved passenger demand will adjust their trip time to take the night last or morning first train according to the train timetable, arriving exactly when the train arrives and experiencing no platform waiting time.

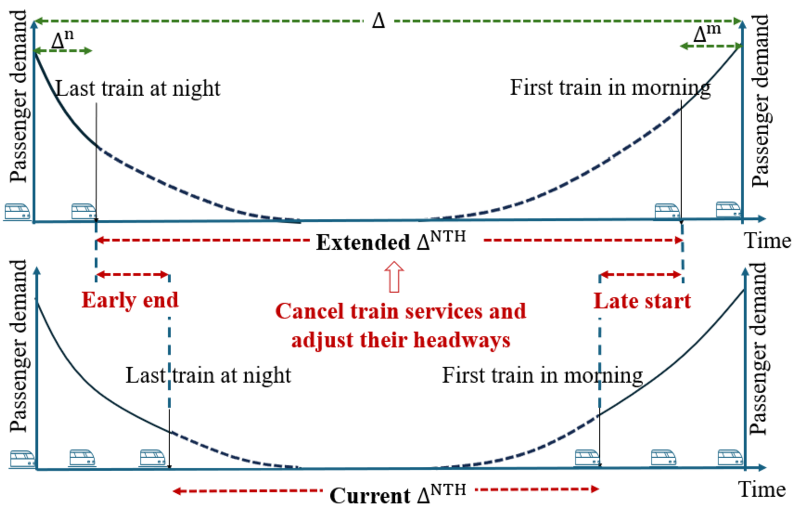

Figure 1 – *Adjustments on night and morning train services with extended requirement for NTH maintenance window*

We consider three components of net profit (NP) in train operation: (1) passenger profit, the profit gained from serving all passenger trips; (2) train operation costs, the costs for running all train services; and (3) passenger waiting time costs, the cost for the platform waiting of all passenger trips. For late night services, the night net profit (NNP) is $NNP = \hat{\theta}(\hat{f} - C_{sc})(\hat{\zeta} + (1-\hat{\zeta})\int_0^{\hat{T}_N} Y(t)\,dt) - NC_{nc} - \hat{\theta} VOT \sum_{j \in S_n} \int_{\hat{T}_{j-1}}^{\hat{T}_j} (\hat{T}_j - t) Y(t)\,dt$. For morning train services, the morning net profit is $MNP = \bar{\theta}(\bar{f} - C_{sc})(\bar{\zeta} + (1-\bar{\zeta})\int_{\bar{T}_M}^{\Delta} Z(t)\,dt) - MC_{nc} - \bar{\theta} VOT \sum_{k \in S_m} \int_{\bar{T}_k}^{\bar{T}_{k-1}} (\bar{T}_{k-1} - t) Z(t)\,dt$. We assume late-night passenger arrivals follow an exponential distribution, $Y(t) = \hat{\lambda} e^{-\hat{\lambda} t}$, and we can prove the night net profit (NNP) is concave with respect to all $\hat{H}_j$, $\forall j \in S_n$. We then assume early-morning passenger arrivals follow a distribution $Z(t) = \bar{\lambda} e^{-\bar{\lambda}(\Delta - t)}$, and we can prove that the morning net profit (MNP) is concave with respect to all $\bar{H}_k$, $\forall k \in S_m$.

3 RESULTS

3.1 Analytical results

First, we optimize night or morning train services separately without considering Δ^{NTH}.

Proposition 1 *If the passenger arriving follow an exponential distribution at night, the number of passengers boarding a train is equal to the number of passengers arriving at the time this train departs, multiplied by the headway of the following train under the optimal train timetable schedule.*

The optimal timetable is achieved when the first derivative of NNP with respect to $\hat{H}_j, \forall j \in S_n$ is zero. Then we obtain the optimal headway of the last and non-last trains: $\hat{H}_N = \frac{1}{\hat{\lambda}} \ln(1 + \frac{\hat{\lambda}}{VOT}(\hat{f} - C_{sc})(1 - \hat{\zeta}))$ and $\hat{H}_j = \frac{1}{\hat{\lambda}}(e^{\hat{\lambda}\hat{H}_{j-1}} - 1), j \in S_n, j \neq N$.

Proposition 2 *If the passenger demand follow an exponential distribution at night, the optimal train headway for the last night train is not affected by the number of train services. This means that the night service will end once the train headway reaches* $\frac{1}{\hat{\lambda}} \ln(1 + \frac{\hat{\lambda}}{VOT}(\hat{f} - C_{sc})(1 - \hat{\zeta}))$.

We define a sequence of variables $\alpha_j = \hat{\lambda}\hat{H}_{N+1-j}$, to represent the scaled train headway, and $\alpha_1 = \alpha$. These variables are indexed in the reverse order from that of train services, and thus is in the backward sequence of time. $\alpha_1 = \ln(1 + \frac{\hat{\lambda}}{VOT}(\hat{f} - C_{sc})(1 - \hat{\zeta}))$. Using Taylor' expansion, we obtain the recurrence relation $\alpha_j \approx \alpha_j - \frac{1}{2}\alpha_j^2 + \frac{1}{3}\alpha_j^3$ and the general formula $\alpha_j = \alpha - \frac{j-1}{2}\alpha^2 + \frac{(j-1)(3j-2)}{12}\alpha^3$. Therefore, we obtain the optimal number of night train services: $N = \frac{1}{\alpha} + \frac{1}{3} - \frac{2\hat{\lambda}C_{nc}}{\hat{\theta}VOT\alpha^3}$. We set a feasible range of train services, denoted as $[N_{min}, N_{max}]$. The lower limit (N_{min}) ensures the basic level of service, while the upper limit (N_{max}) is constrained by the train rolling stock capacity. If a N obtained from $\delta(N) = 0$ is not an integer, the feasible solution must be the minimum integer that is greater than N, i.e., $N^* = \lceil N \rceil$.

Proposition 3 *If passenger demand follow "mirrored" exponential distribution in the morning, the number of passengers boarding a train is equal to the number of passengers arriving at the time when it departs, multiplied by the headway of its following train under the optimal train schedules.*

The optimal timetable is achieved when the first derivative of MNP with respect to $\bar{H}_k, \forall k \in S_m$ is zero. Then we obtain: $\bar{H}_M = \frac{(\bar{f} - C_{sc})(1 - \bar{\zeta})}{VOT}$ and $\bar{\lambda}\bar{H}_{k-1} = 1 - e^{-\bar{\lambda}\bar{H}_k}, \quad k \in S_m, k \neq M$.

Proposition 4 *If the passenger arrivals in the morning follow an "mirrored" exponential distribution, the optimal train headway for the first morning train is not affected by the number of train services. This means that the morning service will start once the scheduled train headway reaches* $\frac{(\bar{f} - C_{sc})(1 - \bar{\zeta})}{VOT}$.

We define a sequence of scaled headways, denoted as $\beta_k = \bar{\lambda}\bar{H}_{M+1-k}$, $\forall k \in S_m$. β_k is indexed in the forward sequence of time, while the train headways are indexed in the reverse sequence of time. Then, we denote $\beta = \beta_1$. Using Taylor's expansion, we obtain the general formula $\beta_k = \beta - \frac{k-1}{2}\beta^2 + \frac{(k-1)(3k-4)}{12}\beta^3$, and the optimal number of morning train services $M = \frac{1}{\beta} + \frac{2}{3} - \frac{2\bar{\lambda}C_{nc}}{\theta VOT\beta^3}$. In practice, M would be an integer, and thus the minimum integer that is greater than M.

Second, if we consider NTH maintenance window in timetable optimization, we should jointly optimize night and morning train services. While obtaining numerical solutions is relatively easy due to the small number of train services during these periods, deriving analytical solutions is challenging due to the complex constraints involved. Therefore, we will elaborate on how to perform joint optimization of the two periods numerically.

3.2 Numerical results

We use the smart card transaction data from individual passengers on metro line 2 in Chengdu, China in September 2021. Specifically, we extract data from passengers who entered the URT systems between 22:00–23:00 at night on September 11th and 6:00–7:00 on September 12th. When the two periods are optimized separately, the optimal number of train services for night is 8 and for morning is 5. The last night train has a headway of 6.7 minutes and the first morning train service has a headway of 7.1 minutes. In the case, the night train services ends at 22:46, and starts at 6:29 in the early morning. If the Δ^{NTH} exceeds the current available time period between 22:46 and 6:29, cancelling on night or/and morning train services are essential. Figure 2 shows the optimal number of night and morning train services corresponding to different Δ^{NTH} values. Generally, for every cancellation of a night train service, there is a corresponding cancellation in the number of morning train services. Additionally, the number of morning train services is first reduced to the minimum requirement of one train service.

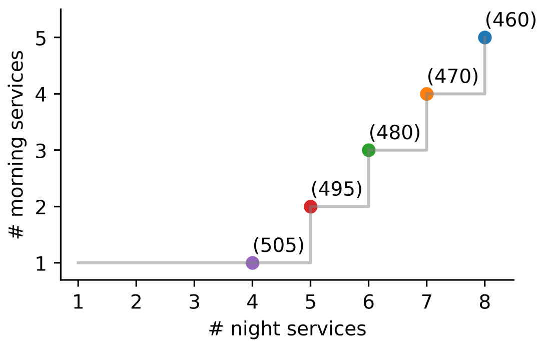

Figure 2 – *Optimal number of night and morning train services with respect to* Δ^{NTH} *with some representative* Δ^{NTH} *values marked*

4 DISCUSSIONS

This study establishes single-line model for optimizing train service schedules of the first few and the last few trains, while also meeting the requirement of NTH maintenance window. It offers a novel perspective on coordinating late-night and early-morning passenger demand, maintenance work, and system operation. The single-line model yields valuable insights into the URT system operation and identifies the trend with evolving requirement for NTH maintenance window. There are some promising directions for future research. One important area to explore is extending our models to a network level, which can provide a more comprehensive understanding of optimization. By addressing this issue, we can improve the effectiveness of our optimization models and ensure a smooth implementation in real-world cases.

References

MTR Corporation Limited. 2023. *MTR putting in over $65 billion to enhance railway maintenance in the next five years adopting innovation and technology to implement smart railway asset management.*