Matching and Reallocation Problem of Reserved Parking Considering Unpunctuality: A Two-Phase Approach

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Keywords: Reservation matching, Unpunctuality, Rolling optimization, Operational management

1 INTRODUCTION

Parking affects daily travel and mobility heavily, and also, changes public travel, urban land use, and transport system (Xu et al. 2021). According to Shoup (2006), cars cruising for parking created 30% of traffic congestion in road networks. To address these issues, a more efficient parking system needs to be designed. Recent developments have enabled a series of innovations and possibilities in dealing with parking operation and management, e.g. parking reservation. Existing studies, e.g., Lei and Ouyang (2017) have considered parking reservation and allocation mechanism with implicit assumption that users will strictly follow their reservations. However, in practical parking operation management, due to the uncertainty in travel time and other unexpected disturbances, these parking serving times would be different from requests that registered in the platform. The unpunctuality could cause parking serving conflicts, exacerbating parking service, and leaving parking space idle and resource wastage. Currently, there is limited research addressing these issues. Existing studies propose flexible reservation mechanisms from a planning perspective (Wang and Wang, 2019), and utilize uncertain planning methods to address parking reservation and allocation problems (Wang et al. 2022), with the consideration of early arrival or late departure.

From the perspective of operational management, this paper designs a two-phase reservation matching and real-time reallocation framework for short-term parking under the assumptions that distributions of drivers' arrival and departure deviation index have been learned and drivers' real-time location sharing. Considering early/late arrival and departure, this framework can be applied to short-term parking reservation systems facing supply-demand imbalances, aiming to better utilize scarce parking resources. There are two main challenges in building the framework, i.e. (i) which requests to accept and (ii) how to reallocate the requests that have been successfully matched to address the issues caused by unpunctuality. Through effective control and reallocation, the parking system is expected to better deal with unpunctuality and reduce the impact of risks on matching. The main objective of this paper is to propose a reservation matching and real-time reallocation framework for short-term parking, aiming to address issues caused by drivers' unpunctuality. The main contributions are summarized as follows:

1. The reservation matching and reallocation problem of reserved parking is raised considering unpunctuality, and a two-phase framework is proposed to address it. To determine which requests to be accepted during the reservation phase, a 0-1 quadratic programming **(0-1 QP)** model is proposed to maximize the net profit of parking system, which is based on distributions of drivers' arrival and departure deviation index and that the service failure rates are calculated.
2. The Phase-type (PH) distribution is used to fit characteristics of drivers' arrival and parking time distribution, which is based on real short-term parking lot data form Beijing and further applied in the numerical experiment. The ensemble of the PH distribution is dense in the domain of all positive distributions, meaning it can be used to approximate any positive distribution.
3. A real-time rolling horizon framework is employed for reallocating the requests with confirmed reservations during the service phase, which aims to minimize the service failure rate and is based on real-time drivers' location and space occupancy information.

2 METHODOLOGY

We consider an intelligent parking reservation system with a set of parking spaces $\mathcal{S}$ and public parking requests $\mathcal{R}$, where $|\mathcal{S}| = N$ and $|\mathcal{R}| = M$. The entire service phase is uniformly divided into several time interval segments using l_1 as the standard (e.g., $l_1 = 15\ min$). The set of intervals is denoted as $\mathcal{K} = \{1,2, \dots, k, \dots, K\}$. The intelligent parking reservation system mainly consists of a historical parking data storage center, a real-time parking request location information center, a real-time parking space occupancy information center, and a reservation matching and reallocation center. For each parking space $s \in \mathcal{S}$, it can be reserved throughout the entire parking service phase. For each parking request $r \in \mathcal{R}$, a binary variable q_{rk} is introduced, where q_{rk}=1 indicates that the reserved parking time window of request r includes interval k, and 0 otherwise. To represent the matching result, a binary indicator x_{rs} is introduced, and is defined to be 1 if request r is matched with space s, and 0 otherwise. We can get the parking request matrix $Q = (q_{rk})_{M\times K}$ and matching result matrix $X = (x_{rs})_{M\times N}$, $r \in \mathcal{R}, k \in \mathcal{K}, s \in \mathcal{S}$.

For each request r, its reserved parking time window is $[a_r, d_r]$, while its actual parking time window is $[a_r{}', d_r{}']$. Then, the arrival and departure deviation index of each request r is denoted as U_r^a and U_r^d, where $U_r^a = a_r{}' - a_r$ and $U_r^d = d_r{}' - d_r$.Utilizing the data from the historical parking data storage center, the distribution of U_r^a and U_r^d can be learned. Then, $U_r^a \sim f_r(\delta)$ and $U_r^d \sim g_r(\delta)$. If the request r arrives within $[a_r, a_r + l_1]$ or departs within $[d_r - l_1, d_r]$, it is defined as normal parking, and early/late arrival and departure, otherwise. For each request r, the service failure rate P_r represents the probability that the system fails to serve it. The calculation expression is as follows:

$$P_r = \sum_{s\in\mathcal{S}} \sum_{r'\in\mathcal{R}} x_{rs} p_{r'r} x_{r's} \tag{1}$$

Where $p_{r'r}$ represents the probability of request r' successfully parking while request r fails to park when the two requests are allocated to any parking space. Its expression is as follows:

$$p_{r'r} = \begin{cases} \Pr\{d_{r'} + U_{r'}^d > a_r + U_r^a\}\Pr\{a_{r'} + U_{r'}^a < a_r + U_r^a\}, & r' \neq r \\ 0 & , r' = r \end{cases} \tag{2}$$

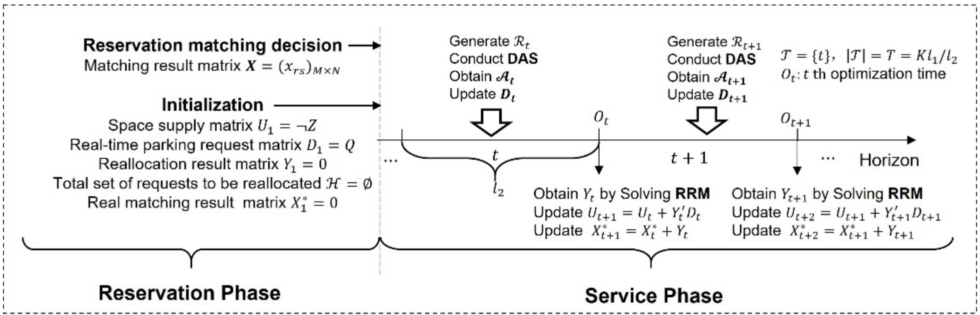


Figure 1 Reservation matching and real-time reallocation framework

To solve the matching problem within the reservation phase, we construct a 0-1 quadratic programming model (**0-1 QP** Model) (3)-(6), which aims at maximizing the net profit of parking system. In equation (3), the first term represents the revenue from accepted requests (the product of unit parking price p_p and total number of reserved time intervals), and the second represents the fine charged for overdue parking requests that can successfully park, and the third represents the penalty paid for failed service requests (the sum of product of unit compensation price p_c, service failure rate and total number of reserved intervals for each request).

$$\max_{\mathbf{x}} \pi = p_p \sum_{s\in\mathcal{S}} \sum_{k\in\mathcal{K}} z_{sk} + p_c \sum_{r\in\mathcal{R}} (1 - P_r) I_r - p_c \sum_{r\in\mathcal{R}} P_r \sum_{k\in\mathcal{K}} q_{rk} \tag{3}$$

Equation (4) represents the expected total number of intervals that each request exceeds the reserved time window. Equation (5) stipulates that each parking request can be matched with at most one parking space, while equation (6) limits each parking space to accepting at most one request per interval. Based on the parking request Q and matching result matrix X, we can define the parking space occupancy matrix $Z = (z_{sk})_{N\times K}, k \in \mathcal{K}, s \in \mathcal{S}$, where $z_{sk} = \sum_{r\in\mathcal{R}} x_{rs} \times q_{rk}$. If parking space s is occupied within interval k, then $z_{sk} = 1$, otherwise, $z_{sk} = 0$.

$$I_r = \lceil (max\{E(U_d^r), 0\} - min\{E(U_a^r), 0\})/l_1 \rceil, \quad r \in \mathcal{R} \tag{4}$$

$$\sum_{s\in\mathcal{S}} x_{rs} \le 1, \quad r \in \mathcal{R} \tag{5}$$

$$z_{sk} \le 1, s \in \mathcal{S}, k \in \mathcal{K} \tag{6}$$

$$x_{rs} \in \{0,1\} \tag{7}$$

At the t th optimization period, we define the state of reserved requests $\mathcal{R}_t \equiv \mathcal{R}$.

$$\mathcal{R}_t = \{R_1(t), R_2(t), R_3(t), R_4(t), R_5(t), R_6(t), R_7(t)\} \tag{8}$$

Where $R_1(t) = \{r :$ request r depart within t and $d_r' < d_r - l_1\}$, $R_2(t) = \{r :$ request r is parking within t and $d_r \le (t-1)l_2\}$, $R_3(t) = \{r :$ request r arrives within $t+1$ and $a_r + l_1 \le (t-1)l_2\}$, $R_4(t) = \{r :$ request r arrives within $t+1$ and $tl_2 - l_1 \le a_r < tl_2 - l_2\}$, $R_5(t) = \{r :$ request r arrives within $t+1$ and $a_r \ge tl_2 - l_2\}$, $R_6(t) = \{r :$ request r should arrive before tl_2 but not arrive, and $a_r = tl_2 - l_1\}$, $R_7(t) = \{r :$ request r shouldn't arrive and hasn't arrived within $t+1$ or is parking without exceeding the reserved time window or normally departs within t and $d_r - l_1 < d_r' < d_r$ or has departed or failed to park before $t\}$. An example of dynamic adjustment strategy (**DAS**) is illustrated in Figure 2, where $l_1 = l_2$. Based on $\mathcal{A}_t$, D_t and U_t, a rolling horizon reallocation model (**RRM**) within t th optimization period can be formulated, aiming at maximizing the number of successful matchings, $\sum_{s\in\mathcal{S}}\sum_{r\in\mathcal{A}_t} y_{rs}$. The constraints of **RRM** include $\sum_{s\in\mathcal{S}} y_{rs} \le 1, \quad r \in \mathcal{A}_t, \sum_{r\in\mathcal{R}} y_{rs}\, d_{rk} \le u_{sk}, s \in \mathcal{S}, k \in \mathcal{K}$ and $y_{rs} \in \{0,1\}$.

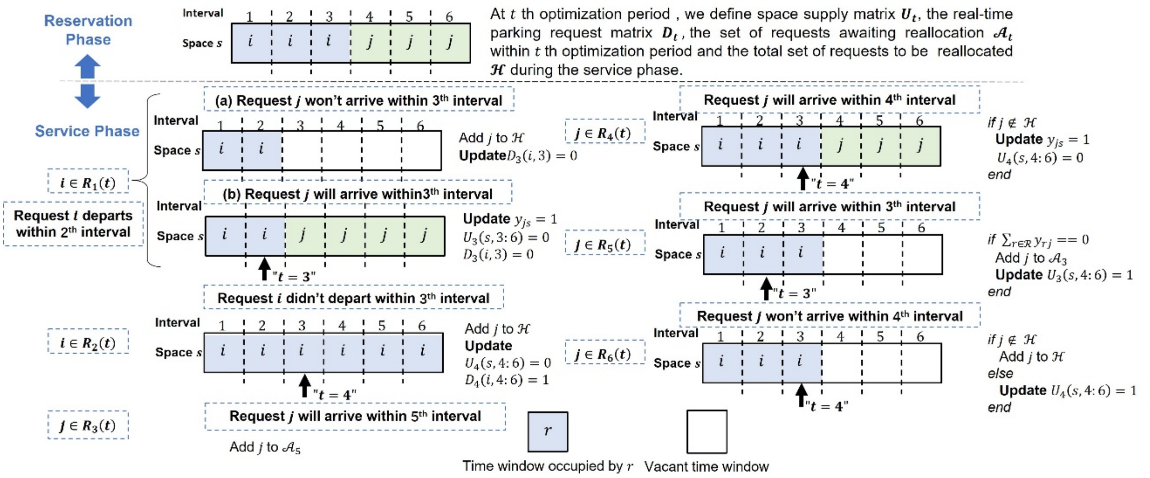


Figure 2 An example of dynamic adjustment strategy

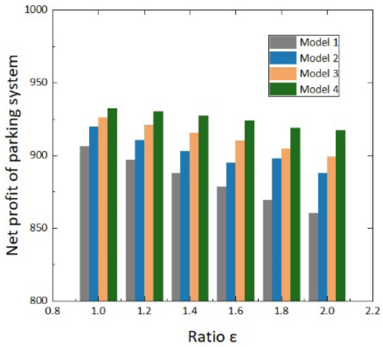
3 RESULTS

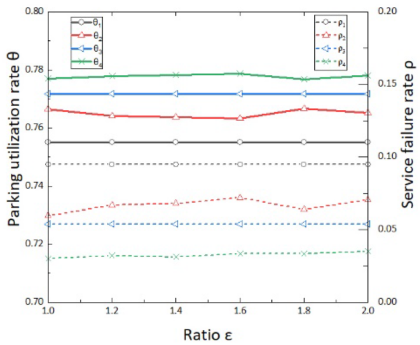
The service phase is set from 10:00 AM to 12:00 AM with $l_1 = l_2 = 15\,min$ and $K = T = 8$. The range of the ratio between the penalty and the profit parameter, $\varepsilon = p_c/p_p$, is set to $[1, 2]$. The number of parking spaces and parking requests are set to $N = 150$ and $M = 350$. The deviation indicators U_r^a and U_r^d respectively follow a normal distribution, expressed as $U_r^a \sim N(\mu_r^a, (\sigma_r^a)^2)$ and $U_r^d \sim N(\mu_r^d, (\sigma_r^d)^2)$. According to the fitting results of a real parking reservation system data in Beijing, we set $\mu_r^a, \mu_r^d \sim N(0,1)$ and $\sigma_r^a, \sigma_r^d \sim U(5,15)$. The drivers' arrival and parking time follow the PH distribution, based on real parking data from a certain CBD parking system in Beijing. Figure

3 illustrates that among the four models, the model proposed in this paper, Model 4, achieves the highest net profit and parking utilization rate while maintaining the lowest service failure rate.

Table 1 Model settings

	Model 1	Model 2	Model 3	Model 4
Reservation phase	Maximize the revenue	Maximize the net profit	Maximize the revenue	Maximize the net profit
Service phase	No operation	No operation	Reallocation	Reallocation





(a)Net profit of parking system (b)Service failure rate and parking utilization rate

Figure 2 Comparison of reservation and allocation models

4 DISCUSSION

In order to address service failure and resource wastage caused by the drivers' early/late arrival and departure, this paper proposes a two-phase reservation matching and real-time reallocation framework. In the reservation phase, a **0-1 QP** model is constructed to decide which requests to accept, with the objective function of maximizing the net profit. In the service phase, requests with confirmed reservations are reallocated in real time under a rolling optimization framework. In the numerical experiment, PH distribution is used to model the arrival distribution of drivers and parking time, based on real short-term parking lot data from Beijing. The results demonstrate that, compared to basic models, the proposed model not only increases the net profit of parking system and parking utilization rate but also reduces service failure rate. We will conduct numerical experiments to explore the scientific validity of the proposed framework and identify optimal settings. The framework can be applied to intelligent parking reservation system, addressing the issues caused by unpunctuality, thereby improving system efficiency. Shared parking is an important management mode to alleviate parking difficulties. In the future, the two-phase framework could be considered for application in shared parking, further addressing conflicts between owners and parkers.

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